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Microeconomic Exercise CES

This document presents a set of exercises in microeconomics as preparation for the SDGSIM Short Training Course for the High Commission for Planning of Morocco.

The CES (Constant Elasticity of Substitution) functional form is widely used in the CGE literature. Analytically, the cost minimization problem of a firm that produces with a CES production technology can be written as

$$\min wL + rK$$

$$\text{subject to } q = \varphi \left(\delta_L L^{\frac{\sigma-1}{\sigma}} + \delta_K K^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where w (r) is the labor (capital) remuneration of, L (K) is the quantity of labor (capital) demanded by the firm, q is the level of production, σ is the elasticity of substitution between labor and capital, and δ_L , δ_K and φ are the share and efficiency parameters of the CES production function. We assume constant returns to scale; i.e., that $\delta_L + \delta_K = 1$.

Exercises

- (1) Analytically define the elasticity of substitution between labor and capital, σ ,
- (2) Graph isoquants when $\sigma = 0$ and $\sigma = \infty$, and
- (3) Derive the following formulas for the labor and capital demand functions from the cost minimization problem that is stated in the introduction – note that λ is the Lagrangian multiplier associated with the production function constraint.

$$L = \left(\frac{\lambda}{w} \right)^{\sigma} \delta_L^{\sigma} \varphi^{\sigma-1} q$$

$$K = \left(\frac{\lambda}{r} \right)^{\sigma} \delta_K^{\sigma} \varphi^{\sigma-1} q$$

- (4) Assuming that $w = r = 1$, $q = 100$, $K = 30$, $L = 70$, and $\sigma = 0.25$,

(a) what are the values of δ_L , δ_K and φ that are consistent with that data? and

(b) how are the results are modified if $\sigma = 2.5$?

HINT: The formulas to calculate δ_L and δ_K can be obtained from the ratio of the first two first order conditions. As you will see, the elasticity of substitution σ shows up in the formulas for δ_L and δ_K .

(5) Starting from the factor demand functions, derive the formulas for the price elasticities of demand for each of the two factors