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A GEM for Streamlined Dynamic CGE Analysis

Structure, Interface, Data, and Macro Application

Martín Cicowiez
Hans Lofgren

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Abstract

This paper provides an overview, macro application, and detailed documentation of GEM-Core, a dynamic computable general equilibrium (CGE) model designed for medium- and long-run policy analysis. GEM stands for General Equilibrium Model. GEM-Core can address the issues that typically are relevant for CGE analysis for developing countries, including fiscal space (with its spending, tax, and foreign aid aspects), public investment, social safety nets, trade, jobs, demography, poverty, and inequality. The model is a template model in the sense that, given an appropriately formatted database, applications for different countries can quickly be developed. The data needed for macro applications are very limited, making it possible to apply the model on short notice to virtually any country, including fragile and low-income countries. GEM-Core comes with a user-friendly Excel-based interface through which the analyst may choose between alternative country databases and, for the selected database, do the analysis (define and analyze simulations, including adjustment of selected data and assumptions). The interface lowers entry barriers to CGE modeling and provides a platform around which training may be organized, making it possible to focus courses on economics instead of computer programming.

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A GEM for Streamlined Dynamic CGE Analysis: Structure, Interface, Data, and Macro Application

Martín Cicowiez and Hans Lofgren*

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* Martín Cicowiez is at the Center of Distributive, Labor and Social Studies (CEDLAS), Universidad Nacional de La Plata, Argentina. Hans Lofgren is in the Development Prospects Group of the World Bank. The authors would like to thank the Strategic Research Program of the World Bank for funding. They can be reached at martin@depeco.econo.unlp.edu.ar and hanslofgren01@outlook.com, respectively.

1. Introduction

The purpose of this paper is to introduce GEM-Core, a dynamic computable general equilibrium (CGE) model designed for medium- and long-run policy analysis. GEM stands for General Equilibrium Model. GEM-Core is a core model in the sense that it can address the issues that typically are relevant for CGE analysis for developing countries, including fiscal space (with its spending, tax, and foreign aid aspects), public investment, social safety nets, trade, jobs, demography, poverty, and inequality. The model may be applied to databases with different disaggregations, ranging from highly aggregated to highly disaggregated. It may be extended when this is needed for special topics. GEM-Core comes with a user-friendly Excel-based interface, ISIM (which stands for "I simulate").[1] It permits the analyst to choose between alternative country databases and, for the selected database, develop an application, something that involves choosing from a menu of pre-programmed assumptions, adjusting selected data, and defining and analyzing simulations. The interface lowers entry barriers to CGE modeling and provides a platform around which training may be organized, making it possible to focus courses on economics instead of computer programming.

The fact that the model may be applied to relatively aggregate databases means that it can be applied on short notice to virtually any country, including fragile and low-income countries. In a companion paper, Cicowiez and Lofgren (2017a) show how the core component of such a database, a macro SAM, may be constructed at low cost from cross-country data; that paper is accompanied by empirical macro SAMs for 133 countries.

In outline, this paper first provides a non-technical overview of GEM-Core and a macro database for an archetype developing country using a SAM from the companion paper (Section 2). After this, it presents and analyzes a set of simulations that demonstrate the use of the model with this macro database (Section 3), winding up with some concluding

[1] The interface is linked to the model program, written in GAMS (the General Algebraic Modeling System; see www.gams.com). Both are available on request from the authors. GEM-Core draws heavily on MAMS (Maquette for Millennium Development Goal Simulations; Lofgren et al. 2013) for which the IFPRI Standard Model provided the starting point (Lofgren et al. (2002). Other country CGE models in GAMS include Décaluwé et al. (2013) and McDonald (2015). The interface developed for MAMS has been adapted to work with GEM-Core. An extension of GEM-Core, named GEM-Trade, has been developed for analysis of preferential trade agreements (Cicowiez and Lofgren 2016).

remarks (Section 4). Appendix A has a full mathematical statement while Appendix B introduces the interface and Appendix C presents additional simulation results in tabular form.

2. Non-technical overview of model structure and database

GEM-Core is a single-country recursive-dynamic general equilibrium model designed for medium- and long-run policy analysis. It is a multi-purpose model in the sense that it can analyze policies in a wide range of areas including growth, fiscal space, and external shocks. It can also be applied to databases with different disaggregations. A typical simulation period is 5-20 years. However, the period is highly flexible, ranging from comparative statics to very long periods, and should be determined by the purpose of the analysis.

This section provides a non-technical overview of GEM-Core and presents its macro database, structured to meet the needs of the model.[2] (A detailed mathematical statement of GEM-Core is found in Appendix A.)

This presentation assumes that the model is applied to a database based on the macro SAM in Table 2.1. For illustration, we use data for a representative (or archetype) low-income country for 2015.[3] [Cicowiez and Lofgren (2017a), provide details regarding the building and interpretation of the SAM in Table 2.1.] The disaggregation of the accounts of the SAM defines the disaggregation of the database and the model application, i.e., it is here disaggregated into two activities and commodities (private and government), two factors of production (labor and private capital), and three institutions (household, government, and rest of world). As indicated by the SAM, each institution has not only current but also capital accounts, something that makes it possible for the model to cover issues related to the financing of current activities and investment. The SAM also depicts the structure of taxation via a set of disaggregated tax accounts.

[2] GEM-Core is part of GEM suite, a set of models with a common core and extensions in selected areas. The model is a descendant of MAMS (Lofgren et al. 2013).

[3] More specifically, the data shown are the weighted average of macro SAMs for the 17 low-income countries for which sufficient information was available in cross-country databases. Each SAM was weighted by its share of GDP at market prices at current US dollars for the countries included.

Table 2.1a. Macro SAM for archetype low-income country in 2015 (percent of GDP)

	act-prv	act-gov	com-prv	com-gov	f-lab	f-cap	hhd	gov	row	tax-act	tax-com	tax-imp	tax-exp	tax-dir	cssoc	cap-hhd	cap-gov	cap-row	invng	invg	dstk	total
act-prv			153.1																			153.1
act-gov				11.7																		11.7
com-prv	63.6	8.4					80.8		19.8										14.4	4.8	-6.2	185.6
com-gov								11.7														11.7
f-lab	49.1	3.3							0.1													52.6
f-cap	40.0								0.2													40.2
hhd					52.3	37.8		1.6	2.8													94.5
gov							1.8		0.9	0.3	5.7	1.4	0.0	4.0	0.0							14.1
row			25.3		0.2	2.4	0.5	0.1														28.5
tax-act	0.3																					0.3
tax-com			5.7																			5.7
tax-imp			1.4																			1.4
tax-exp			0.0																			0.0
tax-dir							4.0															4.0
cssoc					0.0																	0.0
cap-hhd							7.4											0.4				7.7
cap-gov								0.8								2.5		1.5				4.8
cap-row									4.9							-0.1						4.7
invng																11.6		2.9				14.4
invg																	4.8					4.8
dstk																-6.2						-6.2
total	153.1	11.7	185.6	11.7	52.6	40.2	94.5	14.1	28.5	0.3	5.7	1.4	0.0	4.0	0.0	7.7	4.8	4.7	14.4	4.8	-6.2	

Table 2.1b. Accounts in Macro SAM for archetype low-income country in 2015

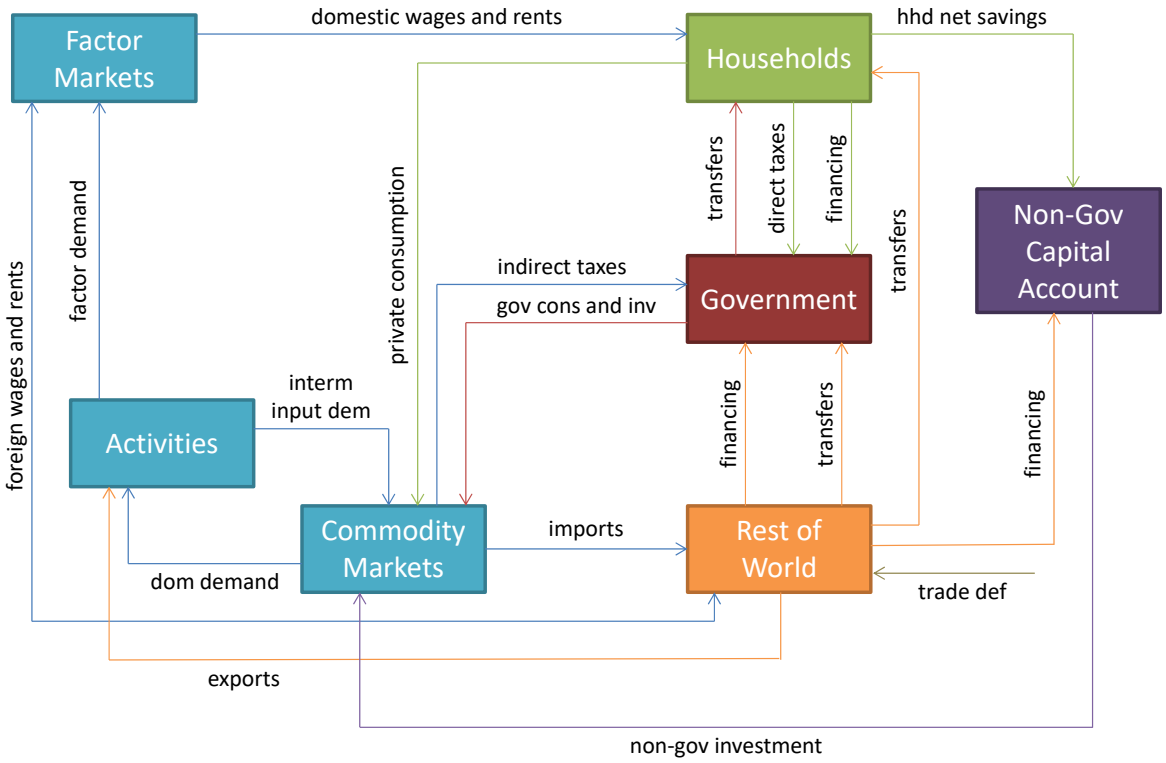
Account	Explanation
act-prv	activity - private production
act-gov	activity - government production
com-prv	commodity - private production
com-gov	commodity - government production
f-lab	factor - labor
f-cap	factor - private capital
hhd	household
gov	government
row	rest of world
tax-act	taxes - activities
tax-com	taxes - commodities
tax-imp	taxes - tariffs on imports
tax-exp	taxes - exports
tax-dir	taxes - income
cssoc	social security contributions
cap-hhd	capital account - household
cap-gov	capital account - government
cap-row	capital account - rest of world
inv-prv	investment - private capital
inv-gov	investment - government capital
dstk	stock change

Source: Cicowiez and Lofgren (2017a).

Model structure

In any single year, GEM-Core has the structure summarized in Figure 2.1. As indicated by the figure, which serves as the reference point for this model overview, the major building blocks of the model are activities (the entities that carry out production), commodities (activity outputs and/or imports; linked to markets), factors (also linked to markets), and institutions (households, enterprises, the government, and the rest of the world). Given the relatively detailed treatment of the financing of private investment (compared to most other CGE models), the private (non-government) capital account also has its own box. In the following model presentation, we assume that the different blocks have the disaggregation presented in the above SAM (Table 2.1).

Figure 2.1. Overview of GEM-Core



Source: Authors' elaboration.

As indicated by Figure 2.1, activities produce and sell their output. According to the SAM, all government output is sold at home while private output is both sold at home and exported. The activities use their revenues to cover costs of intermediate inputs and factors as well as, for the private activity, tax payments. The only factor used by the government is labor while the private activity uses both labor and private capital. For the government, the output level is in effect determined by government demand, a policy tool, which in its turn determines labor hiring and intermediate input demand. For the private sector, profit maximization drives decisions regarding factor employment, which determine the output level and intermediate demands.[4] The split of private output between exports and domestic sales depends on relative sales prices in these two destinations.

[4] In terms of production technology, for both sectors, at the top level of the production nest, intermediate input demand and aggregate factor demand are fixed coefficients per unit of output. At the lower level, the substitutability between labor and capital for the private sector is determined by a CES (Constant Elasticity of

The household earns incomes from factors, transfers from the government, and transfers from the rest of the world. After paying direct taxes (determined by policy), the household spends in fixed shares on private commodity consumption and savings. After deducting net financing of the government and of changes in foreign reserves, household savings are used to finance private investment.

The government gets its receipts from taxes, transfers from abroad, and net financing from households and the rest of the world. It uses these receipts for transfers to households, consumption, and investment (to provide the capital stocks required for government services). To remain within its budget constraint, it either adjusts some part(s) of its spending on the basis of available receipts or mobilizes additional receipts of one or more types to finance its spending plans.

In Figure 2.1, imports and exports (payment to/from the rest of the world for commodities) only apply to the private commodity. Foreign wages and rents is the only non-trade payment to the rest of the world. The non-trade payments received from the rest of the world are net transfers and financing to government and the private sector; the latter also includes foreign investment other than FDI. All non-trade payments are typically exogenous projections.

For the government commodity, the price paid for the demand-driven supply quantity depends on the unit supply cost (for labor and intermediate inputs). In the market for the private commodity, a flexible price ensures balance between demands for domestic output from domestic demanders and supplies to the domestic market from domestic suppliers. Part of domestic demands for the private commodity are for imports; the ratio between demands for imports and domestic output depends on the ratio between the demander prices for commodities from these two sources (i.e., the prices demanders pay including relevant taxes and trade and transport margins) – an increase in the import/domestic price ratio lowers the ratio between the demands for imports and domestic output (and vice versa). Similarly, domestic suppliers (the activities) also consider relative prices when deciding on the allocation of their output between the domestic market and exports. For

Substitution) function. The lower level is not relevant to the government; since it only uses one factor, labor, its coefficient is in effect also fixed. See Appendix A for more detail on functional forms.

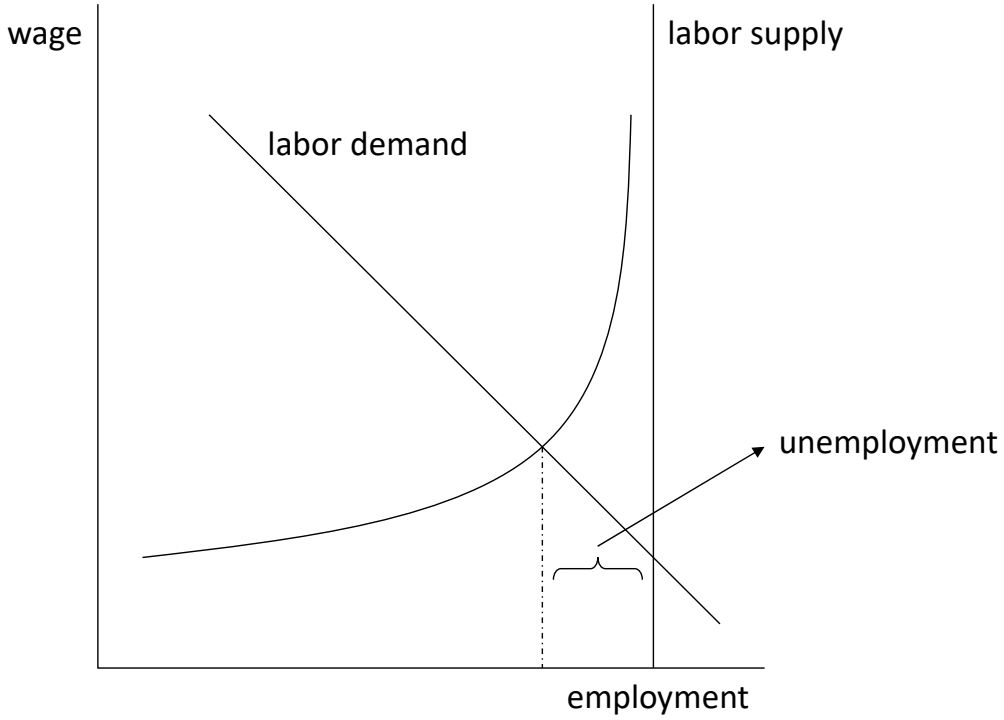
both exports and imports, the standard assumption is that international prices are exogenous (the small country assumption).[5]

These import and export responses to relative price changes underpin the standard clearing mechanism for the balance of payments: changes in the real exchange rate (the ratio between international and domestic price levels, which may change due to changes in the nominal exchange rate) influence export and import quantities and values. For example, other things being equal, an exchange rate depreciation may eliminate a balance of payments deficit by raising the export quantity and reducing the import quantity (and vice versa for an appreciation).

For both labor and capital, the demand curves are downward-sloping, reflecting the production activity responses to changes in wages and rents. For private capital, which within any period has a fixed supply, a flexible rent clears the market. In the labor market, it is typically assumed that unemployment is endogenous with a wage curve that establishes a negative relation between the real wage and the unemployment rate (see Figure 2.2).

[5] Both for imports and exports, the model offers the option of endogenizing prices (in foreign currency) using constant-elasticity demand and supply functions, respectively.

Figure 2.2. Labor market specification



Source: Authors' elaboration.

The above discussion refers to the functioning of the model economy in a single year. In GEM-Core, growth over time is endogenous. The economy grows due to employment growth for private capital and labor as well as growth in total factor productivity (TFP). For capital (only used by the private activity), employment growth coincides with stock growth, which depends on investment and depreciation; for labor, employment growth depends on growth in the stock (which in its turn may be seen as a function of population growth and changes in the labor force participation rates of different age and gender groups) and changes in the unemployment rate. Apart from an exogenous component, the TFP of both the private and the government activity may depend on growth in the public capital stock.

In addition, as noted above, the model covers a set of net financing flows: to the government from domestic non-government institutions (households and enterprises) and the rest of the world; and to domestic non-government institutions from the rest of the world. On the basis of the results for any simulation, assumptions about real interest rates, and initial debt stocks, post-calculations extract the implications for the evolution of

domestic and foreign debt stocks. The same applies to the evolution of the stock of foreign reserves, which is computed on the basis of the initial stock and annual changes.

A model like GEM-Core can help analysts better understand the effects of a wide range of policies and exogenous shocks also when it is applied to a two-sector macro database (the current case). To exemplify, in the fiscal area, it may address the space for government consumption and investment spending under alternative scenarios for TFP growth, foreign aid, and taxation, considering budgetary and sustainability constraints. Alternatively, it may consider the need and consequences of financing a planned spending program from different sources (foreign and domestic). Beyond the fiscal area, it may be used to assess the consequences of shocks affecting world (export and/or import) prices, migration flows, and worker remittances (current private transfers). It is straightforward to address demographic issues, including the impact on growth and living standards of changes in population size and age structure (endogenous or migration driven) and/or changes in age- and gender-specific labor force participation rates. On the other hand, in a macro application, there is little scope for addressing issues related to income distribution and structural change in sectoral production structure. Poverty analysis would have to be done in the context of an unchanged (or exogenously changed) aggregate income distribution. In some cases, it may be helpful to explore the broader macro consequences of scenarios defined by parallel micro analysis that, for example, proposes policy packages with components related to spending, income distribution, and/or factor productivity.

Poverty Module

To compute the poverty and distributional effects of each scenario, GEM-Core implements a poverty module based on MAMS (Lofgren et al., 2013). The module offers a choice between the following approaches: (i) constant elasticity of poverty with respect to per-capita welfare for each model household; (ii) log-normal distribution of per-capita welfare within each model household; and (iii) distribution of per-capita welfare within each model household follows a real-world household survey. In this macro application of GEM-Core, we use approach (ii). The module is linked to base-year poverty and distributional data for each of the representative households (RHs; one or more) in the database. It uses either household income or consumption as its welfare measure. The ability of the module to account for distributional change and its impact on poverty depends on the degree of

disaggregation of the RHs. In applications with a single RH (like the current one), it projects poverty outcomes on the assumption that distribution does not change.[6]

Data

The database for GEM-Core consists of a SAM complemented by data related to factor employment, factor and population stocks, elasticities, and a GDP projection. For this macro application, we used the SAM of Table 2.1, expressed in current US dollars for the aggregate of low-income countries instead of shares of GDP. The role of the SAM is to define base-year values for the bulk of the model parameters, including those covering production technologies, sources of commodity supplies (domestic output or imports), commodity demands (for household and government consumption, investment, stock change, and exports), transfers between different institutions, and tax rates.

The disaggregation of the rest of the database coincides with that of the SAM; i.e., its size is very modest when GEM-Core is applied to a two-sector SAM. The non-SAM database consists of (a) base-year government and private employment; (b) base-year unemployment rate; (c) base-year private and government capital stocks and depreciation rates; (d) three elasticity values (for transformation of output between exports and domestic sales; and for substitution between labor and capital in private production and between imports and domestic output in domestic demand); and (e) projections for the simulation period for population, labor force participation, and GDP growth. This information is readily accessible from standard sources. Table 2.2 shows a-d for our application to an archetype low-income country; Table 2.3 shows e. For instance, we see that the rate of population growth is projected to decline gradually over time, from 2.7 percent in 2016 to 2.4 percent in 2030.

In the base simulation, labor productivity is a free variable, making sure that the projected path of GDP growth is replicated; for the non-base simulations, GDP growth is endogenous. As discussed below in Section 3, for each simulation, it is also necessary to define (a) equilibrating mechanisms for the government budget, the balance of payments, and the

[6] GEM-Core may also feed data to a separate microsimulation module to generate poverty and distributional indicators.

savings-investment balance; and (b) rules for (non-equilibrating) payments in the government budget and the balance of payments.

Table 2.2. Elasticities and base-year factor data for GEM-Core applied to macro SAM

Item	Private	Government	Nation
Elasticities			
factor substitution	0.7		
import - domestic output substitution in demand	1.5		
export - domestic sales transformation in supply	1.5		
TFP with respect to trade openness	0.1		
Employment (%)	95.1	4.9	100.0
Unemployment rate (%)			5.5
Capital stock (GDP share, %)	180.2	65.0	
Capital depreciation rate (%)	4.0	2.5	
Poverty headcount ratio at $1.90 a day* (% of population)			46.2
Gini index			0.428

*2011 purchasing power parity.

Sources: (a) Elasticities: Authors' assessment based on literature; for a survey, see for example Annabi *et al*. (2006, esp. pp. 23-29 and 30-31), finding values in the range 0.3-0.9 for factor substitution and 0.5-2.0 for the two trade-related elasticities; elasticity of TFP with respect to trade openness: Dessus et al. (1999, pp. 27-29). (b) Employment and unemployment: UN (2015), Baddock et al. (2015), and World Bank (2017). (c) Capital stocks: IMF (2017) and authors' assessment. (d) Depreciation: Agénor et al. (2005, pp. 31 and 54; the authors use these rates for Ethiopia). (e) Poverty and inequality: World Bank (2017).

Table 2.3. Projections for GDP, population, and labor force for GEM-Core applied to macro SAM

	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Growth in GDP at factor cost (%)		4.13	4.49	4.79	4.88	4.94	4.99	5.01	5.01	5.01	5.01	5.01	5.01	5.01	5.01	5.01
Population growth (%)		2.71	2.69	2.67	2.64	2.62	2.60	2.57	2.55	2.52	2.50	2.47	2.45	2.42	2.40	2.37
Population 15-64 years (%)*	53.8	54.0	54.2	54.5	54.8	55.1	55.3	55.6	55.8	56.1	56.4	56.7	56.9	57.2	57.5	57.8
Labor force participation rate (%)**	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5	79.5

*Population aged 15-64 years as percent of total population.

**Total labor force as percent of population aged 15-64.

Note: The simulation period is 2015-2030.

3. Simulations

The simulations are designed to demonstrate the types of issues that can be analyzed in a GEM-Core application to a macro database. The first section discusses the base scenario, the second section the non-base scenarios.

Base scenario

The base scenario represents a business-as-usual projection without policy changes. Drawing on projections from IMF's World Economic Outlook (IMF 2017), we impose an average growth rate of 4.9 percent for the period 2016-2030 – this figure corresponds to the projected average growth rate of low-income countries. In the base scenario, GDP growth is imposed by endogenously adjusting labor productivity.[7] We assume that government demand for government services, transfers from government to households, and domestic and foreign government net financing are all maintained at their base-year shares of GDP. Taxes are fixed at their base-year rates, which means that they will grow roughly at the same pace as the overall economy.

At the macro level, GEM-Core – like any other CGE model – requires the specification of equilibrating mechanisms ("closures") for three macroeconomic balances: government, savings-investment, and the balance of payments. For the base scenario, the following closures are used: (a) government: its accounts are balanced via adjustments in the direct tax rate; (b) savings-investment: household savings adjust to generate exogenous GDP shares for domestic private investment while foreign (private) investment is financed via the balance of payments and government investment is covered within the government budget; and (c) balance of payments: the real exchange rate equilibrates this balance by influencing export and import quantities and values; the non-trade-related payments of the balance of payments (transfers and non-government net foreign financing) are non-clearing, kept fixed as shares of GDP.

For each simulation, base and non-base, GEM-Core provides the evolution over time for a wide range of indicators including: (a) macro outcomes: GDP at market prices (split into

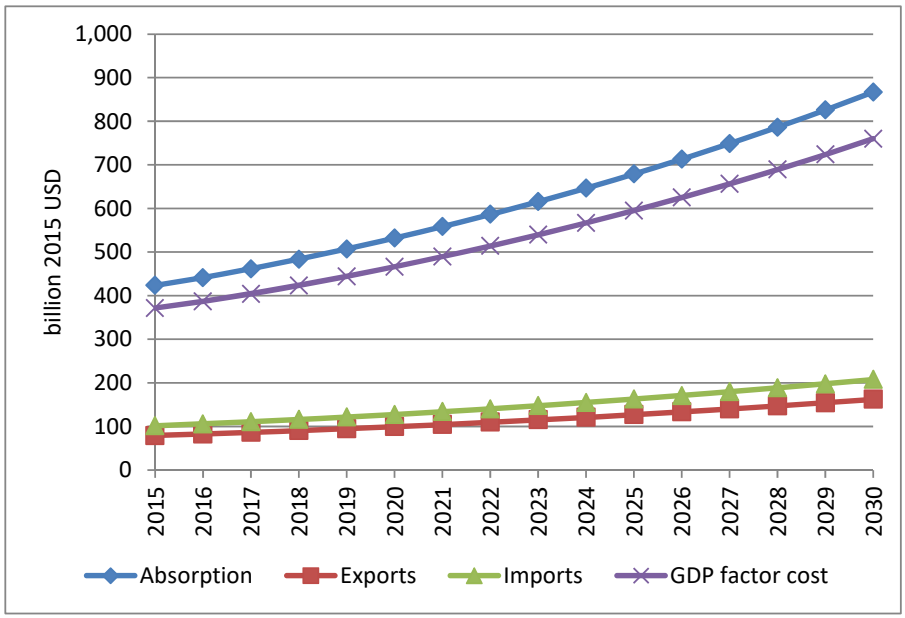
[7] In the non-base simulations, labor-specific productivity is invariably exogenous.

private and government consumption and investment; exports; imports); the composition of the government budget, the balance of payments, and the savings-investment balance; total factor productivity; domestic and foreign debt stocks; (b) sectoral structure of production, value added, incomes, exports, and imports; and (c) the labor market: wages, unemployment, and employment by sector.

Figures 3.1-3.5 show key macroeconomic results for the base. (Tables C.1-C.5 show additional results for base and non-base scenarios, covering macro and sector indicators as well as the government budget and the balance of payments.) Given that it is intended to be a business-as-usual scenario, the base is set up to maintain the initial macroeconomic structure. Figures 3.1 and 3.2 show the evolution of the levels of GDP, foreign trade, and domestic final demand aggregates; in Figure 3.3, this information is translated into average annual growth rates, which are near identical across the different indicators (at 5.0-5.1 percent per year during the period 2018-2030). At the sector level, the GDP growth rates are very close to aggregate GDP growth (Figure 3.4).[8] In the background, the real exchange rate depreciates slightly over time (see Table C.1). Domestic government debt increases from 18 to 26 percent of GDP whereas the foreign debt of the government stays roughly unchanged at around 30 percent of GDP (Table C.2). GDP growth is strong enough to reduce the unemployment rate, from 5.4 percent in 2017 to 4.3 percent in 2030. The real wage grows at a rate of 1.7 percent per year on average. In per-capita terms, household consumption grows at a rate of 2.4 percent per year, leading to a significant decrease in the poverty rate, from 44.6 in 2017 to 30.2 in 2030 (Figure 3.5).

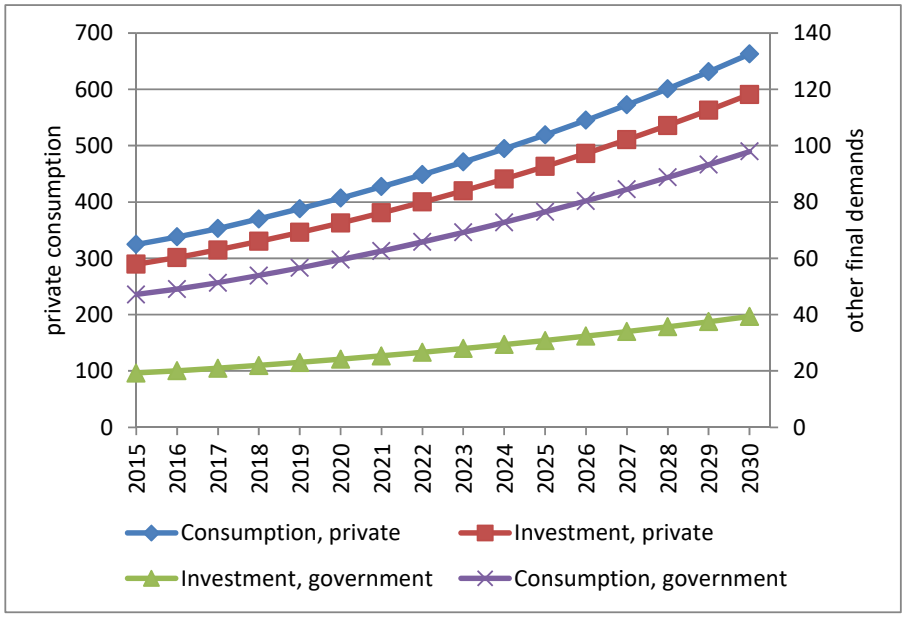
[8] The marginally faster growth for government services is due to a decline in their relative price, leading to higher real government consumption growth in the base setting with fixed spending shares for the components of absorption. In its turn, this relative price decline reflects relatively high intensity for government services in labor, the factor with positive productivity growth in the context of our procedure for calibration of GDP growth.

Figure 3.1. Base: Selected macroeconomic indicators (billion 2015 USD)



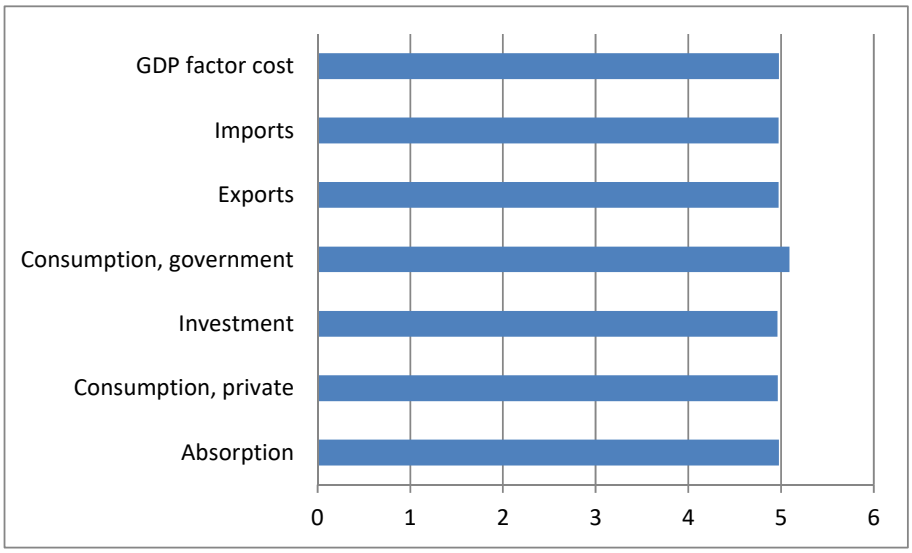
Source: Authors' calculations based on simulation results.

Figure 3.2. Base: Domestic final demands (billion 2015 USD)



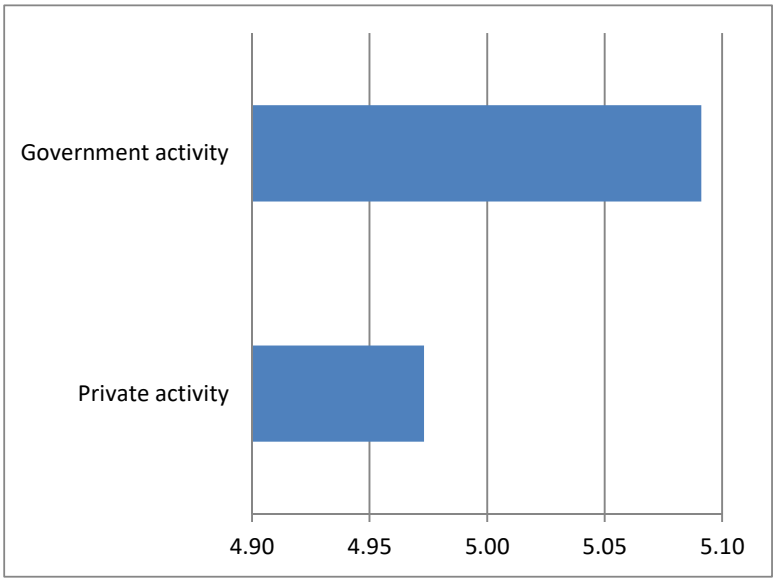
Source: Authors' calculations based on simulation results.

Figure 3.3. Base: Real annual macroeconomic growth 2018-2030 (%)



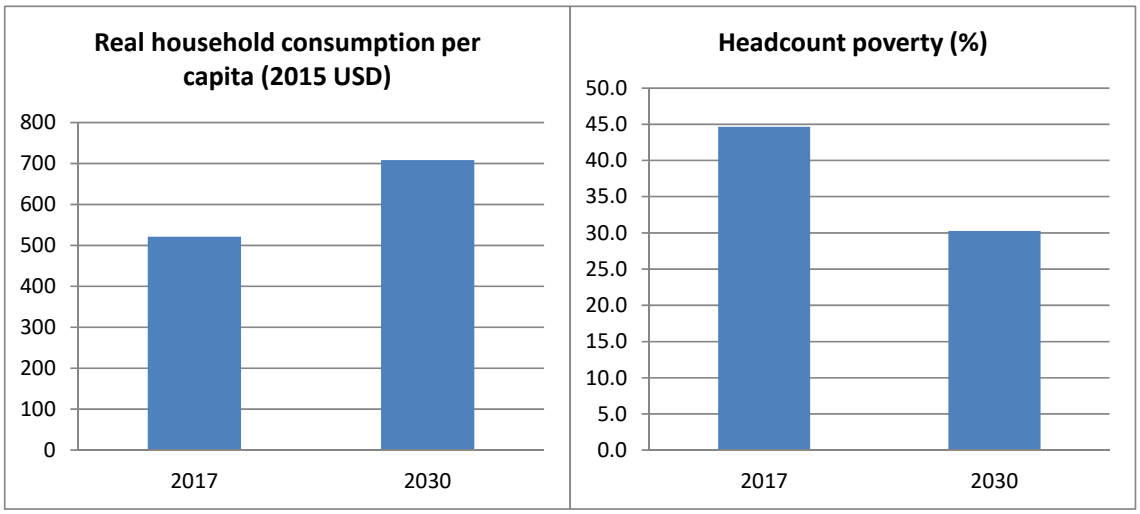
Source: Authors' calculations based on simulation results.

Figure 3.4. Base: Real annual sector growth 2018-2030 (%)



Source: Authors' calculations based on simulation results.

Figure 3.5. Base: Real household consumption per capita and headcount poverty



Source: Authors' calculations based on simulation results.

Non-base scenarios

The non-base scenarios are defined in Table 3.1. They demonstrate some of the issues that can be addressed with GEM-Core when it is applied to a two-sector macro database. As shown in Table 3.1, the first three scenarios test the effects of an expansion in government investment, accompanied by an adjustment in one of three alternative financing sources (direct taxes, net domestic financing, or net foreign financing) that clears the government budget. The two remaining scenarios address the effects of external shocks: increases in remittances (transfers to households) from abroad and the world export price.

Table 3.1. Definitions of non-base scenarios*

Name	Description
ginv-tdir	Government investment increase by 2 %-age points of base GDP in 2018-2030; budget cleared via direct tax adjustment
ginv-dbor	Same as ginv-tdir except for that budget is cleared via domestic borrowing adjustment
ginv-fbor	Same as ginv-tdir except for that is budget is cleared via foreign borrowing adjustment
pwe	10.1% increase in the world price for exports during the period 2018-2030
remit	Remittance increase by 2 %-age points of base GDP in 2018-2030

*Note: Except for the changes indicated in the description, the scenarios are identical to the base scenario.

Source: Authors' elaboration.

To facilitate comparisons across the different scenarios, all shocks are imposed during the same period (2018-2030) and are of the same size (in each year, 2 percent of base scenario GDP for the same year). For the scenario *pwe*, the size of the export price shock (10.1 percent) was defined so that, in the absence of changes in export quantities, GDP or the exchange rate, in each year it would be exactly 2% of base GDP. However, for this and all other scenarios, GDP will change and, given this, the shocks (for example the increase in government investment) will not be exactly equal to 2 percent of the GDP of the new scenario.

Apart from the shocks that are imposed (described in Table 3.1), some assumptions are different for the non-base scenarios. For the savings-investment balance, instead of imposing a fixed GDP share for private investment, investment spending (including its GDP share) is endogenous, adjusting to make use of available financing in the context of exogenous household savings rates. For the government balance, the treatment is the same as for the base (with a flexible direct tax rate) except when the clearing variable is changed as part of the simulation design – this will be discussed further below. However, the treatment of the balance of payments is the same as for the base – the real exchange rate clears. Beyond the macro balances, the non-base scenarios also differ from the base in that the following payments are fixed at the levels generated by the base scenario (instead of being fixed as shares of GDP): domestic government financing (fixed in domestic currency, implicitly indexed to the CPI, the model numéraire, as explained in Appendix A); private and government transfers and financing from the rest of the world (fixed in foreign currency).[9]

[9] For the base scenario, imposing GDP shares has the advantage of generating a balanced evolution of targeted indicators. However, for non-base scenarios (which will be compared to the base and to each other), it is not reasonable to assume that, for example, in response to changes in the exchange rate or GDP, payments in foreign currency automatically are adjusted sufficiently to stay unchanged as shares of GDP. Fixing these payments in foreign currency has the additional advantage of leveling the playing field across the different simulations – they are to an identical extent able to rely on payments from the rest of the world and, unless otherwise noted, the level of foreign liabilities is identical at the end of the simulation period.

It is important to note that the above-mentioned changes in assumptions for the non-base simulations have been introduced following an approach that assures that, if there were no other shocks (like those defined in Table 3.1), the base results would be exactly replicated. This is achieved via adjustments of parameters for the non-base scenarios (related to the institutional payments and the macro balances) on the basis of the base results. However, as intended, when other shocks are introduced, then these changes have an impact on the results; for example, the impact of higher remittances on GDP growth is different depending on whether the level of private investment is a fixed GDP share, or is determined by the amount of domestic private financing for investment.[10]

Government Investment Scenarios

Firstly, we assess the impact of an increase in government investment, under alternative financing mechanisms. For the government investment scenarios -- *ginv-tdir*, *ginv-dbor*, and *ginv-fbor* – the increase is 2 percent of base GDP in each year; this generates an increase in annual government investment growth for the period 2018-2030 from 5.0 to 7.9 percent (Table C.1). A likely motivation for higher government investment is that the government capital that is created raises factor productivity. In the model, changes in government capital stocks is one of the determinants of sectoral TFP (the value of the efficiency terms in sectoral value-added functions), thus providing a link between government investment and TFP. The strength of the link is determined by sector-specific elasticities of TFP with respect to the ratio between current and base-year government capital stocks. Drawing on data from the literature, the elasticity is set to generate a total marginal product (measured in real value added) of 0.125 for government capital, i.e., other things being equal, one additional dollar of government capital raises total value added by 0.125. In the absence of any information to the contrary, the elasticity is the same for the private and government sectors. Following the back-of-the-envelope procedure described in Lofgren and Cicowiez

[10] More concretely, the base scenario generates a path for household savings rates that is consistent with the private investment GDP shares that are imposed. For all non-base scenarios, the path of household savings rates is defined using base results while the private investment GDP share now is endogenous. If this were the only change introduced in a non-base scenario, then the results would be the same as for the base. However, if shocks are introduced, then the response will be different when private investment is savings-driven as opposed to having an exogenous GDP share (the base assumption).

(2014), we estimated the internal rate of return for government investment at 12.1 percent.[11]

The main results for the government investment scenarios are presented in Figures 3.6-3.11; additional information is found in Tables C.1-C.5. The results show that the impact of increased government investment very much depends on the financing mechanism. Compared to the base, GDP growth increases substantially when marginal financing comes from direct taxes (*ginv-tdir*) or foreign borrowing (*ginv-fbor*) but declines when it stems from domestic borrowing, as it reduces the resources available for domestic private investment (*ginv-dbor*; Figure 3.6). If the expansion is financed by additional borrowing, then the 2030 government debt stocks increase significantly, as shares of GDP by around 15 percent for *ginv-fbor* and by 22 percent for *ginv-dbor*; the larger increase for the latter scenario is primarily due to slower GDP growth (Table C.2). In addition to the financing source, the payoffs from government investment obviously depend on the TFP responses to changes in the government capital stock; thus, the relevant elasticities should be scrutinized, considering the specific country context (including the quality of public investment management and the extent to which the kinds of investments that are expanded are bottlenecks to growth); sensitivity analysis is also warranted to better understand the impact of alternative elasticity values and, by extension, the importance of public investment policy making and management. Among other macro indicators, the gains in absorption are particularly high for *ginv-fbor* (where the trade deficit increases as exports and imports respond to the appreciation of the real exchange rate). As expected, for the different components of absorption (Figure 3.7), the main growth deviation from the base is for government investment. (Real government consumption would change only if it is shocked as part of the simulation.) For *ginv-fbor*, private consumption and investment growth increase while they decrease for *ginv-dbor* and remain virtually unchanged for *ginv-tdir*. As a share of GDP, the need for additional financing from these different sources is

[11] In the literature, estimates of the internal rate of return for government investment vary widely; average values may be in the range of 10-20 percent. For example, the median for observations for low-income countries for different investment types reported in Foster and Briceño-Garmendia (2010, p. 71) is 14.3 percent; see also Dessus and Herrera (2000, p. 413). In general, these values would be expected to vary depending on (a) the types of investment; (b) the level of development (i.e., size of initial capital stock relative to economic needs, relatively low at low levels of GDP per capita) (Calderon and Serven, 2014); and (c) the quality of government institutions.

smaller the greater the success of the scenario in raising growth; for example, in 2030, the addition to foreign borrowing is 1.6 percent of GDP for *ginv-fbor* while, for the scenario *ginv-dbor*, additional domestic borrowing amounts to 2.4 percent of GDP (Table C4). In per-capita terms – population growth is exogenous -- the annual growth deviations for private consumption range between gains and losses at close to 0.2 percentage points for the simulations *ginv-fbor* and *ginv-dbor*, respectively (Figure 3.8), which translate into changes in the 2030 poverty rate of around 1 percentage point (Figure 3.9). The GDP changes due to government investment are limited to the private sector since investment goods are produced by the private sector (or imported) while the demand for government services is policy-driven and unchanged (Figure 3.10).

Figure 3.11 provides information on the time path for deviations from the base for private consumption and investment for this set of scenarios. For *ginv-fbor* and *ginv-dbor*, the positive and negative deviations for private consumption from the base gradually expand over time. However, for *ginv-tdir*, an initial loss in private consumption (due to the increase in direct taxes, needed to finance the government investment expansion) diminishes gradually until it is virtually eliminated by 2030 (Figure 3.11); if the level of real government investment in 2030 would have reversed to the base level and direct taxes would have been adjusted downwards, then private consumption would have been 2.1 percent above the base level. This points to the fact that the attractiveness of taxation for growth-enhancing investment depends on time preferences. The picture for private investment is similar: a moderate gain for *ginv-fbor*, a sharp loss for *ginv-dbor*, and full recovery after an initial loss for *ginv-tdir* (Figure 3.11); it is obvious that changes in private consumption and investment in most contexts are highly correlated since household incomes net of direct taxes are spent in fixed shares on consumption and savings, used to finance domestic private investment.

External shocks

The second set of scenarios tests the implications of two external shocks: increases in world export prices and remittances. Figures 3.6-3.10 and 3.12 show results.

In the first scenario, labeled *pwe*, export prices (in foreign currency), are raised so that, other things being equal (including unchanged export quantity, GDP, and exchange rate), the gain in export income would have amounted to 2 percent of GDP in each year 2018-

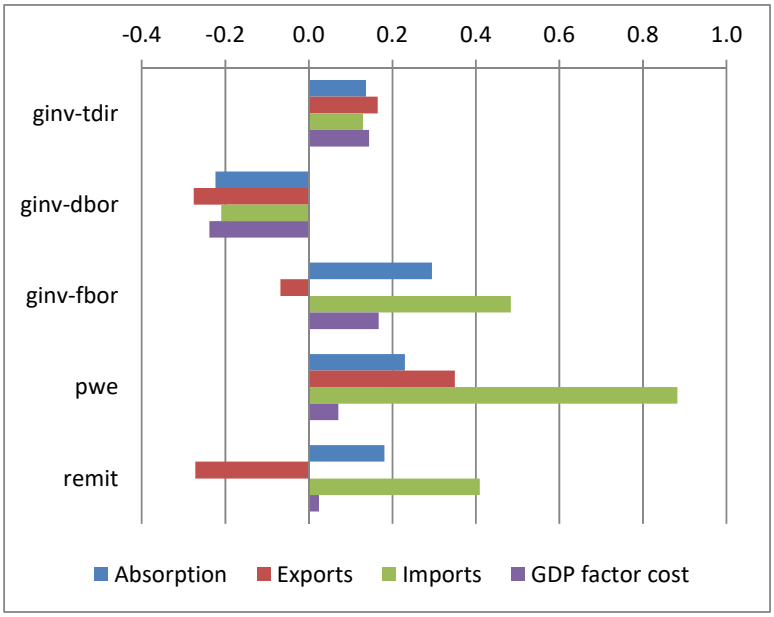
2030; given that exports are around 20 percent of GDP, this translates into a price increase of around 10 percent. As expected, higher export prices lead to an exchange rate appreciation (at an annual rate of 0.5 percent; Table C.1), higher growth in GDP at factor costs, exports, imports, absorption, and private consumption (Figures 3.6 and 3.7). Among these developments, the moderate GDP gain is due to slight increases in private investment and capital stocks as well as an increase in TFP (due to increased openness) while the larger absorption and consumption gains also are boosted by the larger trade deficit. Household per-capita consumption growth accelerates by 0.3 percentage points, leading to a poverty rate decline of around 1.6 percentage point by 2030 (Figures 3.8-3.9). With regard to the time path, for private consumption, a gain of close to 3.7 percent is realized in 2018; it is maintained over time (Figure 3.12).

The relatively feeble gain in private investment growth is due to the fact that the exchange rate appreciation reduces the domestic value of foreign investment (which is exogenous in foreign currency, unchanged from the base scenario level) enough to counterbalance most of the increase in the part of private investment that is financed out of domestic savings. This outcome is sensitive to the specifics of this application, including the share of foreign investment in total private investment (part of the base-year data set), how it evolves over time (including how it responds to changing conditions in the domestic economy), as well as the degree of flexibility in the response of the economy to the positive terms-of-trade shock.

In the second external shock scenario, *remit*, remittances from abroad to the household (which are exogenous in foreign currency) are increased by 2 percentage points of base scenario GDP. The resulting increase in household income gives rise to increased growth for absorption, private consumption, private savings, private investment, and GDP (Figures 3.6 and 3.7). In response to the initial surplus in the balance of payments, the real exchange rate appreciates, bringing about lower export growth, higher import growth, and a larger trade deficit. The appreciation is more moderate than for the scenario *pwe* since the improvement in export incentives is absent. The loss in export growth may be a concern, signaling a mild form of Dutch disease – by 2030, real exports are around 3 percent below the base 2030 level. From another angle, it is the increase in the trade deficit that makes it possible for absorption to grow more rapidly independently of any increase in GDP growth.

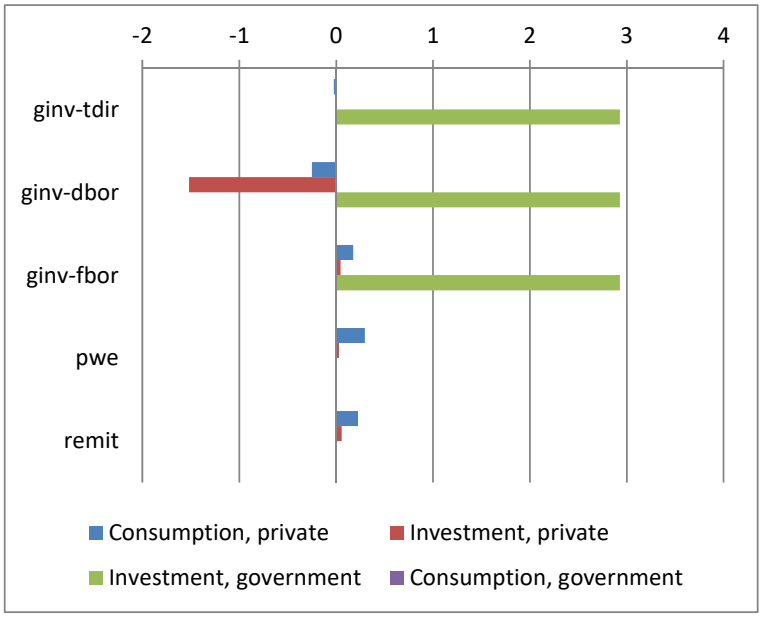
Note also that the extent to which the model splits this increase in the trade deficit (due to the remittance shock) across adjustments in exports and imports depends on economic structure, including the relative responsiveness of exports and imports to exchange rate changes; in the model, the level and relative values of trade elasticities should reflect such structural features (cf. elasticity data in Table 2.2).

Figure 3.6. Macro growth by simulation (%-age point deviation for average annual growth from base)



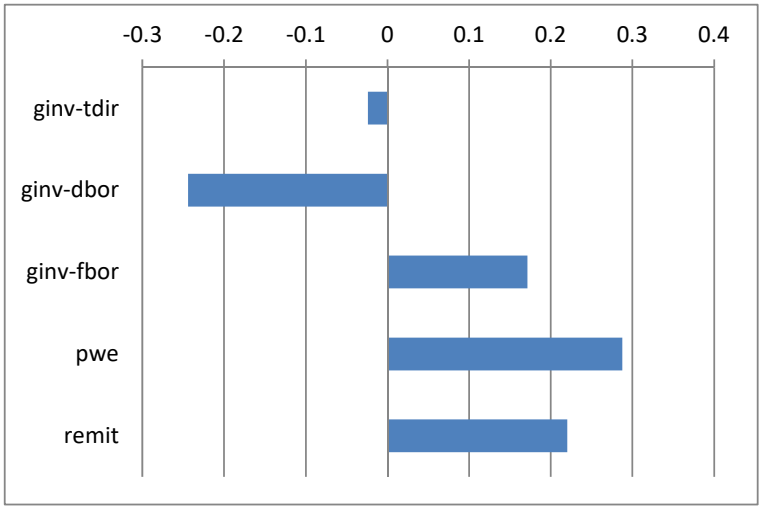
Source: Authors' calculations based on simulation results.

Figure 3.7. Consumption and investment growth by simulation (%-age point deviation for average annual growth from base)



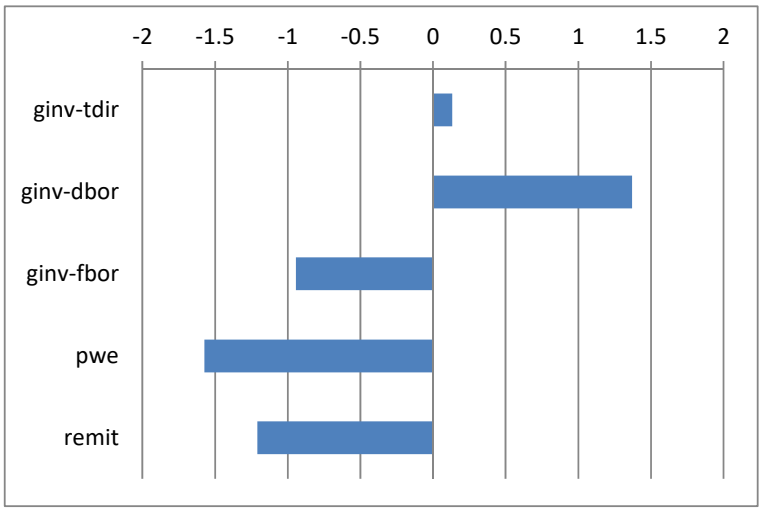
Source: Authors' calculations based on simulation results.

Figure 3.8. Real household consumption per capita growth by simulation (%-age point deviation for average annual growth from base)



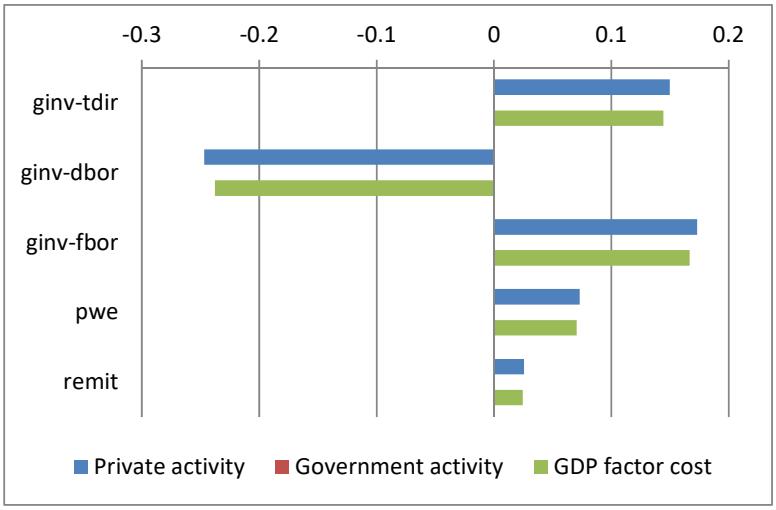
Source: Authors' calculations based on simulation results.

Figure 3.9. Headcount poverty by simulation (%-age point deviation from base)



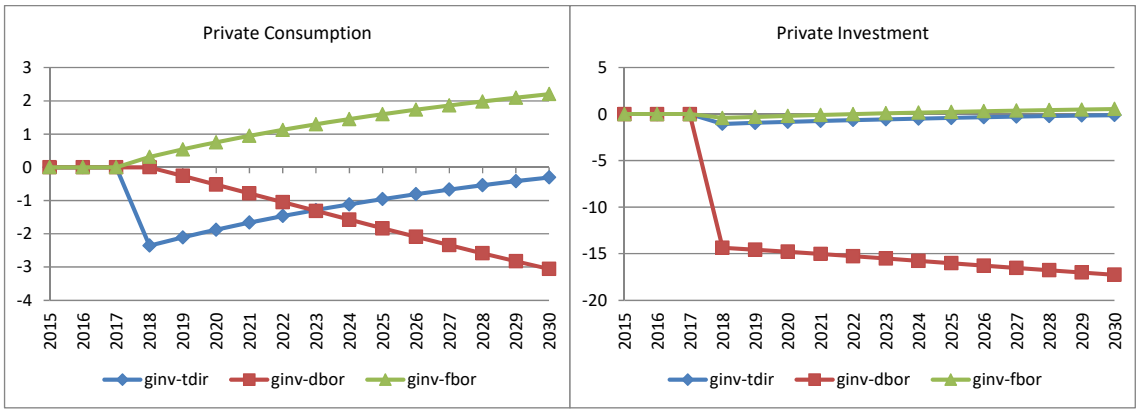
Source: Authors' calculations based on simulation results.

Figure 3.10. Sectoral GDP growth by simulation (%-age point deviation from base)



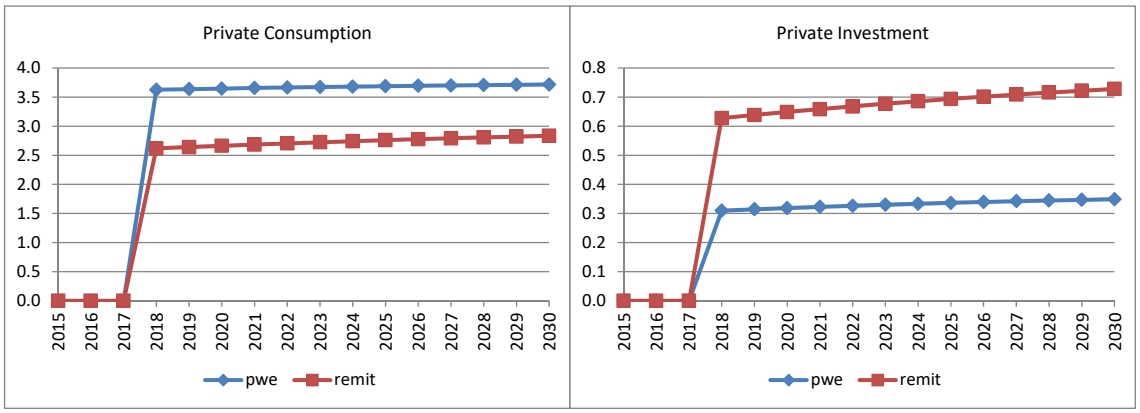
Source: Authors' calculations based on simulation results.

Figure 3.11: Real private consumption and investment for government investment simulations
(% level deviation from base)



Source: Authors' calculations based on simulation results.

Figure 3.12: Real private consumption and investment for external shock simulations
(% level deviation from base)



Source: Authors' calculations based on simulation results.

4. Conclusions

This paper presents GEM-Core, a model for country-level CGE analysis, covering its structure and data needs, and demonstrating its use in a macro application. The simulations in this paper are examples of the types of shocks that can be simulated with a two-sector database. Other examples include macro aspects of taxation, foreign aid, demography, labor force participation, savings, investment, and domestic and foreign debt sustainability.

The Excel-based interface, which has been developed for the model (described in Appendix B), facilitates model applications; however, to make good use of the model, it is essential to understand its structure (presented in Appendix A) and learn from applying it.

A companion paper (Cicowiez and Lofgren, 2017a) provides macro SAMs for 133 countries; these provide the bulk of the data needed for macro applications. Given that these SAMs and the rest of the databases needed for country applications are very small, they can easily be adjusted and updated by the analyst.

However, while macro applications meet some analytical needs and may be preferable for training purposes, GEM-Core and its interface are equally applicable to databases that have finely disaggregated households, factors, and sectors; this is possible thanks to the use of set notation in the model (parameters, variables, and equations) and its database. The degree of disaggregation that is preferable in any applications depends on purpose, data availability, and the time and resources available for the analysis. Given the fact that CGE databases are built from secondary data (i.e., they do not require new surveys) that are available for most countries, the resources needed for the work are relatively limited.

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Appendix A: GEM-Core Mathematical Statement

This appendix presents a mathematical statement of GEM-Core, showing the relationships that, together with the database, determine the results of model simulations. A good understanding of the structure of the model and its database is needed to well understand the simulation results. The appendix is divided into two subsections, notation (A.1) and equations (A.2). Throughout, the presentation is organized around a set of tables.

A.1. Notation

Table A.1.1 explains notational principles, designed to make it easy to understand the statement. Tables A.1.2-A.1.5 define model sets, variables, Latin-letter parameters, Greek-letter parameters, respectively. In each of these tables, the items are arranged alphabetically. Given that this model is dynamic, a time index is part of the domains of all variables and the parameters that are most likely to change over time.

All model components are potentially active but whether they are used in any given application depends on the disaggregation of the database. In Table A.1.2 the right-most column shows the set definitions in the macro application of this paper. Parameters, variables, and equations for which the domains are empty sets are not part of the model and most of those in the model have very few set elements; as a result, the model that is solved is very small. In addition, the sets over which the model items are defined have very few items; as a result, the model that is solved is in fact very small.[12]

Table A.1.1. Notational principles

Items	Notation	Example
Sets	Lower-case Latin letters as subscripts to variables and parameters	see the following rows
Endogenous variables	Upper-case Latin letters (without a bar)*	QG_c

[12] More concretely, as a consequence of the fact that the sets *FOTH, TAC, TACD, TACE,* and *TACM* are empty (Table A.1.2) and in the absence of an explicit subsidy account in the SAM, the model in this application does not include the variables *QT* and *SUBCT* (Table A.1.3), the parameters *icd*, *ice*, and *icm* (Table A.1.4), and the equations TRD13 (Table A.2.2) and INS13 (Table A.2.3). To exemplify the role of few members in most sets, note that, in each year *t*, the variable $QE_{c,t}$ is only defined over one *c*, the private commodity.

Exogenous variables**	Upper-case Latin letters with a bar*	$\overline{QFS}_f$
Parameters**	Lower-case Latin letters* or lower-case Greek letters (with or without superscripts)	$ica_{c,a}$; ρ_c^q

*The names of Latin letter variables and parameters that refer to prices, quantities, and factor wages (rents) start with *P*, *Q*, and *WF*, respectively.
**The distinction between exogenous variables and parameters is that the latter always have exogenous values whereas the former under alternative assumptions may be endogenous.

Table A.1.2. Sets

Name	Description	Elements
$a \in A$	activities (production sectors or industries)	act-prv, act-gov
$c \in C$	commodities (i.e., goods and services)	com-prv, com-gov
$c \in CD(\subset C)$	commodities with domestic sales of domestic output	com-prv, com-gov
$c \in CE(\subset C)$	exported commodities	com-prv
$c \in CM(\subset C)$	imported commodities	com-prv
$c \in CT(\subset C)$	transactions commodities (services paid under distribution margins)	
$d \in D$ $(INSD \cup FCAP \cup A \cup TAC)$	domestic demanders (or demand types): institutions (for consumption), investment by capital type, activities, transactions (distribution margins)	hhd, gov, f-cap, act-prv, act-gov
$f \in F$	factors	f-lab, f-cap, f-capg
$f \in FVA(\subset F)$	factors that earn value added (in SAM)	f-lab, f-cap
$f \in FCAP(\subset F)$	capital factors	f-cap
$f \in FCAPG(\subset FCAP, \not\subset FVA)$	gov't capital factors (do not earn value-added)	f-capg
$f \in FCAPNG(\subset FCAP, \subset FVA)$	non-gov't capital factors (earn value-added)	f-cap
$f \in FLAB(\subset FVA)$	labor factors (earn value-added)	f-lab
$f \in FOTH(\subset FVA,$ $\not\subset FLAB, \not\subset FCAP)$	other factors (earn value-added; not capital or labor)	
$f \in FUEND(\subset FVA)$	factors with endogenous unemployment rate	f-lab
$i \in INS$	institutions	hhd, gov, row

$i \in INSD(\subset INS)$	domestic institutions	hhd, gov
$i \in INSDNG(\subset INSD)$	domestic non-government institutions	hhd
$i \in INSNG(\subset INS)$	non-gov't institutions (rest of world and elements in *INSDNG*)	hhd, row
$h \in H(\subset INSDNG)$	households	hhd
$t \in T$	time periods (simulation years)	2015-2030
$t \in TMIN$	base period (first simulation year)	2015
$tac \in TAC$	transactions (distribution) types (domestic, import, export)	--
$tacd \in TACD(\subset TAC)$	transactions (distribution) for domestic sales	--
$tace \in TACE(\subset TAC)$	transactions (distribution) for exports	--
$tace \in TACM(\subset TAC)$	transactions (distribution) for imports	--

Table A.1.3. Variables

Name	Description
CPI_t	consumer price index
$DKA_{f,a,t}$	change in capital stock *f* allocated to activity *a*
$DKINS_{i,f,t}$	investment by institution *i* (in *INS*) in capital stock *f*
DPI_t	domestic producer price index (PDS-based)
EG_t	total current government expenditure
$EH_{h,t}$	consumption expenditure for household *h*
EXR_t	exchange rate (local currency per unit of foreign currency
$INV_{i,t}$	value of investment (including stock change) for institution *i* (in *INSNG*)
$INVG_t$	value of investment (including stock change) for government
$MPS_{i,t}$	marginal propensity to save for domestic non-government institution *i* (in *INSDNG*)
$MPSSCAL_t$	*MPS* scaling factor
$NFFG_t$	net foreign financing of government (FCU)
$NFF_{i,t}$	net foreign financing for non-government institution *i* (in *INSDNG*) (FCU)
$PA_{a,t}$	output price for activity *a*
$PDD_{c,t}$	demand price for commodity *c* (in *C*) produced and sold domestically
$PDS_{c,t}$	supply price for commodity *c* (in *C*) produced and sold domestically
$PE_{c,t}$	price for export of *c* (in *C*) (LCU) (net of export taxes and distribution margin)

$PK_{f,t}$	price (per unit of) of capital stock *f*
$PM_{c,t}$	price for import of *c* (in *C*) (LCU) (includes import tariffs and distribution margin)
$PQD_{c,d,t}$	composite commodity price for *c* (in *C*) for domestic demander (type) *d* (in *D*) [includes commodity subsidies, all taxes (including VAT and sales tax), and distribution margins]
$PQS_{c,,t}$	composite commodity price for *c* (includes import tariffs and distribution margins but not sales tax, commodity subsidies, or VAT)
$PX_{c,t}$	producer price for commodity *c*
$PVA_{a,t}$	value-added price for activity a
$QA_{a,t}$	level of activity *a*
$QD_{c,t}$	quantity sold domestically of domestic output *c*
$QE_{c,t}$	quantity of exports of commodity *c* (in *C*)
$QF_{f,a,t}$	quantity demanded of factor *f* by activity *a*
$QFINS_{i,f,t}$	endowment of institution *i* (in *INSD)* of factor *f*
$QG_{c,t}$	quantity of government consumption of commodity *c*
$QGSCAL_t$	government consumption scaling factor
$QH_{c,h,t}$	quantity consumed of commodity *c* by household *h*
$QINT_{c,a,t}$	quantity of commodity *c* as intermediate input to activity *a*
$QINV_{c,t}$	quantity of investment demand for commodity *c* (investment by source)
$QINVSCAL_t$	investment scaling factor
$QM_{c,t}$	quantity of imports of commodity *c* (in *C*)
$QQ_{c,t}$	quantity of composite demand (and supply) of commodity *c* (in C)
$QT_{c,t}$	quantity of trade and transport services demand for commodity *c* (in C)
$QX_{c,t}$	quantity of domestic output of commodity *c* (in C)
$RGDPMP_t$	real GDP at market prices (at constant base-year prices)
$SAVF_t$	foreign savings (FCU)
$SAVG_t$	government savings
$SAV_{i,t}$	savings of domestic non-government institution *i* (in *INSDNG*)
$SHIF_{i,f,t}$	share for institution *i* (in *INSD*) in the income of factor *f*
$SUBCT_t$	government spending on commodity subsidies
$TFP_{a,t}$	total factor productivity for activity *a*
$TFPSCAL_t$	scaling of total factor productivity
$TRDGDP_t$	real foreign trade (exports+imports) and GDP ratio
$TRII_{i,i',t}$	transfers to institution i (in *INS*) from domestic non-government institution *i'* (in *INSDNG*)

$UERAT_{f,t}$	unemployment rate for factor *f*
$WALRAS_t$	variable check on Walras' law (which is satisfied if value is zero)
$WF_{f,t}$	economywide wage of factor *f*
$WFAVG_{f,t}$	average wage for factor *f* (in *FCAPNG*)
$WFDIST_{f,a,t}$	wage distortion factor for factor *f* in activity *a*
$YF_{f,t}$	income of factor *f*
YG_t	government current revenue
$YI_{i,t}$	income of (domestic non-government) institution *i* (in *INSDNG*)
$YIF_{i,f,t}$	income of institution *i* (in *INSD*) from factor *f*

Table A.1.4. Latin letter parameters

Name	Description
$capcomp_{c,f}$	quantity of commodity *c* per unit of new capital stock *f*
$cwts_{c,h}$	weight of commodity *c* in consumption basked of household *h*
$depr_{f,t}$	rate of depreciation for capital stock *f*
drf_t	change in foreign reserves (FCU)
$dwts_c$	weight of commodity *c* in the DPI (PDS-based producer price index)
$ica_{c,a}$	quantity of intermediate input *c* per unit of activity *a*
$icd_{c,c'}$	input of *c* for trade and transportation per unit of commodity *c'* produced and sold domestically
$ice_{c,c'}$	transactions input of *c* per unit of commodity *c'* export
$icm_{c,c'}$	transactions input of *c* per unit of commodity *c'* imports
$invshr_{f,i,t}$	share for capital stock *f* in investment spending of institution *i* (in *INSNG*)
$mpsb_{i,t}$	baseline marginal propensity to save for domestic non-gov't institution *i* (in *INSDNG*)
$ndfg_t$	net domestic financing to government (indexed to numéraire) (FCU)
$nff_{i,t}$	net foreign financing to institution *i* (in *INSD*) (FCU)
$pop_{ac,t}$	population of *ac* (household *h* in *H* or country total)
$pwe_{c,t}$	export price for commodity *c* (in foreign currency)
$pwm_{c,t}$	import price for commodity *c* (in foreign currency)
$qdstk_{c,i,t}$	change in stock (inventories) of *c* for institution *i* (in *INSD*)
$qfinsb_{i,f,t}$	endowment for institution *i* (in *INSD*) of factor *f* (in *FOTH*)
$qgb_{c,t}$	baseline quantity of government consumption of commodity *c*
$qg01_{c,t}$	0-1 parameter turning on-off potential scaling of gov consumption of *c*
$qinvb_c$	base-year quantity of investment (GFCF) demand for *c*

$shii_{i,i'}$	share of institution *i* (in *INS*) in the income (net of direct taxes and savings) of domestic non-gov't institution *i'* (in *INSDNG*)
$sub_{c,d,t}$	rate of subsidy on commodity *c* (in *C)* for demander *d* (in *D*)
$ta_{a,t}$	rate of tax on gross output value for activity *a*
$te_{c,t}$	rate of tax on commodity *c*
$tf_{f,t}$	rate of direct tax on factor *f*
$tfpb_{a,t}$	exogenous component of TFP for activity *a*
$tm_{c,t}$	rate of import tariff on commodity *c*
$tq_{c,t}$	rate of sales tax on commodity *c*
$trnsfr_{ac,i,t}$	transfers from institution i (gov't or rest of world) to *ac* [where *ac* is institution *i* (in *INS*) or factor *f* (in *F*)](LCU if from gov't; FCU if from rest of world)
$tva_{c,d,t}$	rate of value-added tax on commodity *c* (in *C*) for demander *d* (in *D*)
$ty_{i,t}$	rate of direct tax on domestic non-gov't institution *i* (in *INSDNG*)
$ueratb_{f,t}$	exogenous unemployment rate for factor *f* (not in *FUEND*)
wfb_f	exogenous economywide wage term for activity-specific factors
$wfdistb_{f,a}$	exogenous activity-specific wage term for mobile factors

Table A.1.5. Greek letter parameters

Name	Description
$\alpha_{i,t}^{sav}$	intercept in savings function for institution *i* (in *INSDNG*)
$\beta_{c,h}$	share parameter in LES function for household consumption of commodity *c*
$\gamma_{c,h}^{min}$	minimum quantity in LES function for household consumption of commodity *c*
δ_c^{dd}	share parameter for domestic purchases in Armington function for commodity *c* (top of nest)
δ_c^{ds}	share parameter for domestic sales in CET function for commodity *c* (top of nest)
δ_c^{e}	share parameter for exports in CET function for aggregated commodity *c* (in *C*) (top of nest)
δ_c^{m}	share parameter for imports in Armington function for commodity *c* (top of nest)
$\delta_{f,a}^{va}$	share parameter for factor *f* in CES VA function for activity *a*
$\eta_{a,f}^{tfp}$	elasticity of TFP in activity *a* with respect to gov't capital stock *f*
η_f^{wf}	elasticity of wage for factor *f* (in *FUEND*) with respect to unemployment rate
$\theta_{a,c}$	yield of output *c* per unit of activity *a*

κ_f	sensitivity of the allocation of new capital for *f* (in *FCAPNG*) across activities (in *A*) to current deviations of activity capital rents from the economywide average
ρ_c^q	exponent in Armington function for commodity *c*
ρ_a^{va}	exponent in CES VA function for activity *a*
ρ_c^x	exponent in CET function for commodity *c* (top of nest)
σ_c^q	elasticity of substitution between supplies of domestic output and imports in Armington function for *c* (top of nest)
σ_a^{va}	elasticity of substitution between factors in CES VA function of activity *a*
σ_c^x	elasticity of transformation between domestic sales and exports in CET function for *c* (top of nest)
φ_c^q	shift parameter in Armington function in which domestic sales and imports of commodity *c* (in *C*) are aggregated to composite supply *(*top of nest)
φ_a^{va}	shift parameter for CES VA function of activity *a*
φ_c^x	shift parameter in CET function for commodity *c (*top of nest)

A.2. Equations

The equations are split into four blocks:

1. Production and factors;
2. Domestic and aggregate foreign trade;
3. Current accounts of domestic institutions
4. Investment, system constraints, and numéraire.

Each section of the presentation covers one block and has its equations stated in one table. In model simulations, it is possible to choose among alternative assumptions for (i) payments linking the government, domestic non-government institutions, and the rest of the world; and (ii) the equilibrating mechanisms (the closures) for macro balances, factor markets, and markets for exports and imports. The assumptions used in this paper are presented in Section 3.[13] In this appendix, we apply the following set of relatively simple assumptions:

- Government budget: The government balance is cleared by adjustments in government investment in the context of rule-based or exogenous levels for other government

[13] The User Guide that accompanies ISIM-GEM-Core provides full details on these and other model features.

payments (including exogenous values for tax rates, quantities of government consumption, and foreign and domestic financing).

- Savings-investment: The level of domestically financed private investment is determined by the level of financing from domestic non-government institutions, for which the marginal propensities to save are fixed. Government investment is financed as part of the government budget.
- Balance of payments: The balance is cleared by adjustments in the real exchange rate, which influence export and import quantities and values; other items in the balance of payments (including transfers, foreign investment, and net foreign financing) are exogenous or determined by other rules.
- Factor markets:
    - Private capital is activity-specific (not mobile across activities), with an activity-specific market-clearing wage.
    - Other factors (including labor) are mobile across activities; unemployment is endogenous for selected factors (typically labor).
- Foreign markets for exports and imports. Both world export and import prices are exogenous (i.e., the small country assumption).

A.2.1. Production and factors

The equations in this block are found in Table A.2.1. They cover the determination of production by sector, demands for factors and intermediates, TFP, factor wages (or rents), unemployment, and factor incomes.

The activity levels (*QA*), which drive the level of commodity production by each activity, is a CES function of factor employment, scaled to account for the contribution of intermediate inputs (PRD1). Factor demands (*QF*) are a function of the parameters of the production function, wages, and the price of value added (i.e., the payment to factors per unit of the activity), in a setting where the producers maximize profits while taking prices and wages as given (PRD2). *TFP* by activity is a function of an exogenous trend parameter, a scaling parameter (which typically is endogenous for the base simulation but otherwise exogenous), ratios between current and base-year government capital stocks, and openness to trade as

the ratio between (a) the sum of real exports and imports; and (b) real GDP (PRD3). (The latter ratio is defined in Table A.2.2.) In this equation, the impacts of government capital stocks and openness to trade are both captured by a constant-elasticity formulation.

Other variables related to production are determined by activity levels, other parameters, and prices. Intermediate demands (*QINT*) are a Leontief fixed-coefficient function of activity levels (PRD4). Likewise, commodity output levels (*QX*) are driven by activity levels multiplied by fixed yield coefficients (θ), summed over all relevant activities (PRD5). Depending on the values of the yield coefficients, any commodity may be produced by more than one activity and any activity may produce more than one commodity. The value-added price (*PVA*), which appeared above in the factor demand functions (PRD2), is defined as the price (or revenue) per unit of an activity (*PA*) net of activity taxes and the intermediate input cost per activity unit (PRD6). For any activity, *PA* is the product of yields and unit producer prices, summed over all outputs (PRD7).

The treatment of factor markets is rich, making it possible for the analyst to select among alternative assumptions with regard to mobility, unemployment, and supply growth. In this mathematical statement, we assume that (i) private capital his fully employed and activity-specific (with endogenous allocations of private capital created by new investment); and (ii) that other factors (labor and natural resources, if any) are mobile and may or may not have endogenous unemployment (depending on a set definition). Other configurations are possible.

Table A.2.1 shows the treatment of the markets for factors other than private capital, which is treated in Table A.2.4 given its links to investment and its special treatment of mobility. For non-labor factors, the unemployment (excess-capacity) rates are fixed (PRD8). For labor, wages are determined by a “wage curve”, which is a function of the base-year wage and the ratios between current and base-year values for the CPI (the numéraire, which in practice does not change) and the unemployment rate, *UERAT*, which is endogenous and raised to a negative elasticity (PRD9). For all factors, the activity-specific wage term (*WFDIST*) is fixed (PRD10) and, irrespective of whether unemployment is endogenous or not, the factor market equilibrium conditions state that total employment equals total supplies adjusted for unemployment (or excess capacity) (PRD11).

Given the above-stated treatment, the factor market equilibrium conditions (PRD11) are cleared via adjustments in the economywide wage variable (*WF*). For factors not in the set *FUEND*, the quantities supplied for employment (the RHS of PRD11) are fixed in any given time period; given this, the full adjustment burden falls on the LHS and the quantities demanded (defined in equation PRD2). For factors in *FUEND* (often labor), the adjustment is shared between the demand and supply sides. For example, for the case of excess demand (in PRD11, LHS is larger than RHS), an increase in *WF* would simultaneously (i) reduce *QF* and the LHS value of PRD11 (via PRD2); and (ii) reduce the unemployment rate (*UERAT* via PRD8), thereby raising the RHS value of PRD11.

Irrespective of the treatment of the markets, the total income for each factor (*YF*), also including private capital, is the product of the two wage terms and quantities employed, summed over all activities, plus net factor transfers (or income) from abroad, adjusted for the exchange rate (PRD12).

Table A.2.1. Equations for production and factors

PRD-1	$QA_{a,t} = TFP_{a,t} \cdot \varphi_a^{va} \left(\sum_f \delta_{f,a}^{va} \cdot QF_{f,a,t}^{-\rho_a^{va}} \right)^{\frac{-1}{\rho_a^{va}}}$	$a \in A, t \in T$	Value added
PRD-2	$QF_{f,a,t} = \left(\frac{PVA_{a,t}}{WF_{f,t} \cdot WFDIST_{f,a,t}} \right)^{\sigma_a^{va}} \cdot \left(\delta_{f,a}^{va}\right)^{\sigma_a^{va}} \left(TFP_{a,t} \cdot \varphi_a^{va}\right)^{\sigma_a^{va}-1} QA_{a,t}$	$f \in FVA$ $a \in A$ $t \in T$	Factor demands
PRD-3	$TFP_{a,t} = tfpb_{a,t} \cdot \overline{TFPSCAL_t} \cdot \prod_{f \in FCAPG} \left(\frac{QFINS_{gov,f,t}}{QFINS_{gov,f}^{00}} \right)^{\eta_{a,f}^{tfp}} \cdot \left(\frac{TRDGDP_t}{TRDGDP^{00}} \right)^{\eta_{a,trdgdp}^{tfp}}$	$a \in A, t \in T$	Total factor productivity
PRD-4	$QINT_{c,a,t} = ica_{c,a} \cdot QA_{a,t}$	$c \in C, a \in A$ $t \in T$	Intermediate demands
PRD-5	$QX_{c,t} = \sum_{a \in A} \theta_{a,c} \cdot QA_{a,t}$	$c \in C, t \in T$	Output
PRD-6	$PVA_{a,t} = PA_{a,t}\left(1 - ta_{a,t}\right) - \sum_{c \in C} PQD_{c,a,t} \cdot ica_{c,a}$	$a \in A, t \in T$	Value-added price
PRD-7	$PA_{a,t} = \sum_{c \in C} \theta_{a,c} \cdot PX_{c,t}$	$a \in A, t \in T$	Activity price
PRD-8	$UERAT_{f,t} = ueratb_{f,t}$	$f \in FVA, f \notin FUEN$ $f \notin FCAPNG$	Exogenous unemploy-ment rates

PRD-9	$WF_{f,t} = WF_f^{00}\left(\frac{\overline{CPI_t}}{CPI^{00}}\right)\left(\frac{UERAT_{f,t}}{UERAT_f^{00}}\right)^{\eta_f^{wf}}$	$f \in FUEND$ $t \in T$	Wage curve
PRD-10	$WFDIST_{f,a,t} = wfdistb_{f,a}$	$f \in FVA$ $f \notin FCAPNG$	Exogenous activity-specific wage term for mobile factors
PRD-11	$\sum_{a \in A} QF_{f,a,t} = \left(1 - UERAT_{f,t}\right) \cdot \sum_{i \in INSD} QFINS_{i,f,t}$	$f \in FVA, t \in T$ $f \notin FCAPNG$	Factor markets
PRD-12	$YF_{f,t} = \sum_{a \in A} WF_{f,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t} + trnsfr_{f,row,t} \cdot EXR_t$	$f \in FVA, t \in T$	Factor income

A.2.2. Domestic and aggregate foreign trade

Table A.2.2 covers the allocation of domestic commodity demands between imports and domestic output and the allocation of domestic output between exports and domestic sales. Equations TRD1-TRD3 are related to prices. In TRD1, the export price received by producers, *PE*, is defined as the world export price, transformed into domestic currency via the exchange rate and adjusted for export taxes and the transactions (trade and transport) cost per unit of exports; the unit transactions cost is defined as the product of an input coefficient (*ice*) and the input price, summed over all inputs. In analogous fashion, equation TRD2 defines the domestic currency import price for demanders, *PM*, on the basis of the world import price, the exchange rate, and import tariffs, in this case with the unit transactions cost added to the price. In both equations, it is assumed that the modeled economy is small; thus, world prices for exports and imports (*pwe* and *pwm*) are exogenous. Equation TRD3 links the demander and supplier prices for domestic output sold domestically, *PDD* and *PDS*: the demander price is defined as the supplier price plus the transactions cost per unit of domestically sold output; as will be discussed below, either of these prices can be seen as the market-clearing price for this category of outputs (cf. equation INV3).

The commodity demand, *QQ*, is a CES aggregation of imports and domestic purchases, named the Armington function after its originator (TRD4); *QQ* is referred to as a "composite" demand given that it is met from different sources. Equation TRD5 defines the

composite demands for commodities that (as opposed to those covered by TRD4) do not have both imports and domestic purchases.

For commodities with both sources, domestic demanders are assumed to minimize the cost of any composite demand quantity subject to the Armington function and subject to the relative prices. The first-order conditions (FOCs) are made up of the Armington function itself (TRD4), and an equation that specifies the optimal demand ratio (*QM/QD*) as a function of the ratio between the prices of domestic output and imports (*PDD/PM*) (TRD6). The composite price *PQS* is implicitly defined by TRD7 given that the other variables in this equation are determined by other relationships. At the composite commodity level, a distinction is made between *PQS* and *PQD*. As shown by TRD8, the distinction is that *PQD* (the price paid by domestic demanders) is adjusted to account for sales taxes, value-added taxes, and subsidies; given that both value-added taxes and subsidies always or often have different rates for different demander categories, *PQD* is disaggregated along this additional dimension, captured by the index *d*.

Turning to the production side, a constant-elasticity-of-transformation (CET) function defines the frontier for allocations of domestic output (*QX*, defined in the preceding section) between exports and domestic sales (*QE* and *QD*, respectively) (TRD9) for outputs that, according to base data, have non-zero values for both destinations. Equation TRD10 defines the equivalent of this transformation for outputs with only domestic sales or only exports.

For outputs with both destinations, producers are assumed to maximize the revenue of any output quantity subject to the CET function and relative prices. The FOCs are made up of the CET function and an equation that specifies the optimal supply ratio (*QE/QD*) as a function of the ratio between the prices of exports and domestic sales (*PE/PDS*) (TRD11)**.** The average producer output price, *PX*, is defined as a weighted average of the prices received for domestic sales and exports (TRD12). (In section A.2.1, *PX* influences production decisions and revenues.) The demand for trade and transport services is a function of real domestic and foreign trade volumes, using a fixed-coefficient formulation (TRD-13). The final two equations in this block define the real trade-GDP ratio and real GDP, which is the denominator in this ratio (TRD-14 and TRD-15).

Table A.2.2. Equations for domestic and aggregate foreign trade

TRD-1	$PE_{c,t} = \left(1 - te_{c,t}\right) EXR_t \cdot pwe_{c,t} - \sum_{c' \in CT} \sum_{tace \in TACE} PQD_{c',tace,t} ice_{c',c}$	$c \in CE$ $t \in T$	Export price
TRD-2	$PM_{c,t} = \left(1 + tm_{c,t}\right) EXR_t \cdot pwm_{c,t} + \sum_{c' \in CT} \sum_{tacm \in TACM} PQD_{c,tacm,t} \cdot icm_{c',c}$	$c \in CM$ $t \in T$	Import price
TRD-3	$PDD_{c,t} = PDS_{c,t} + \sum_{c' \in CT} \sum_{tacd \in TACD} PQD_{c,tacd,t} icd_{c',c}$	$c \in C$ $t \in T$	Domestic demand price for domestic output
TRD-4	$QQ_{c,t} = \varphi_c^q \left(\delta_c^m \cdot QM_{c,t}^{-\rho_c^q} + \delta_c^{dd} \cdot QD_{c,t}^{-\rho_c^q}\right)^{-\frac{1}{\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Composite demand if use of imports and domestic output
TRD-5	$QQ_{c,t} = QM_{c,t} + QD_{c,t}$	$\left(c \in CM \cap c \notin CD\right) \cup \left(c \in CD \cap c \notin CM\right),$ $t \in T$	Composite demand if not use of both imports and domestic output
TRD-6	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \frac{\delta_c^m}{\delta_c^{dd}}\right)^{\frac{1}{1+\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Import-domestic demand ratio
TRD-7	$PQS_{c,t} \cdot QQ_{c,t} = \left(PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}\right)$	$c \in C$ $t \in T$	Composite demand price
TRD-8	$PQD_{c,d,t} = PQS_{c,t}\left(1 + tq_{c,t}\right)\left(1 - sub_{c,d,t}\right)\left(1 + tva_{c,d,t}\right)$	$c \in C, d \in D$ $t \in T$	Adjusted composite demand price
TRD-9	$QX_{c,t} = \varphi_c^x \left(\delta_c^e \cdot QE_{c,t}^{\rho_c^x} + \delta_c^{ds} \cdot QD_{c,t}^{\rho_c^x}\right)^{\frac{1}{\rho_c^x}}$	$c \in CE \cap CD$ $t \in T$	Output transfor-mation if both exports and domestic sales
TRD-10	$QX_{c,t} = QE_{c,t} + QD_{c,t}$	$\left(c \in CE \cap c \notin CD\right) \cup \left(c \in CD \cap c \notin CE\right),$ $t \in T$	Output transforma tion if not both exports and

			domestic sales
TRD-11	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \frac{\delta_c^{ds}}{\delta_c^{e}} \right)^{\frac{1}{\rho_c^x - 1}}$	$c \in CE \cap CD$ $t \in T$	Export-domestic sales ratio
TRD-12	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$	$c \in C$ $t \in T$	Producer output price
TRD-13	$QT_{c,t} = \sum_{c' \in C} \left(icm_{c,c'} \cdot QM_{c',t} + ice_{c,c'} \cdot QE_{c',t} + icd_{c,c'} \cdot QD_{c',t} \right)$	$c \in CT$ $t \in T$	Trade and transport margin demands
TRD-14	$TRDGDP_t = \frac{\sum_{c \in C} EXR^{00} \cdot pwe_c^{00} \cdot QE_{c,t} + \sum_{c \in C} EXR^{00} \cdot pwm_c^{00} \cdot QM_{c,t}}{RGDPMP_t}$	$t \in T$	Real trade-GDP ratio
TRD-15	$RGDPMP_t = \sum_{\substack{c \in C \\ h \in H}} PQD_{c,h}^{00} \cdot QH_{c,h,t} + \sum_{\substack{c \in C \\ f \in FCAP}} PQD_{c,f}^{00} \cdot capcomp_{f,c} \cdot \sum_{i \in INS} DKINS_{i,f,t} + \sum_{c \in C} PQD_{c,dstk}^{00} \cdot \sum_{i \in INS} qdstk_{c,i,t} + \sum_{c \in C} PQD_{c,gov}^{00} \cdot QG_{c,t} + \sum_{c \in C} EXR^{00} \cdot pwe_c^{00} \cdot QE_{c,t} - \sum_{c \in C} EXR^{00} \cdot pwm_c^{00} \cdot QM_{c,t}$	$t \in T$	Real GDP at market prices

A.2.3. Current payments by domestic institutions

This equation block explains payments that are part of the current accounts of domestic institutions, i.e. current incomes and spending for households, the government and enterprises. In the model and its database, it is necessary to include at least one household, and, in practice, models applied to countries in this world invariably have a government. Enterprises are optional. Even though the model and the database can handle multiple representative households, this mathematical statement assumes for simplicity that there is only one household. The sets for institutions distinguish between *INSD* (all domestic institutions), *INSDNG* (all non-government domestic institutions, i.e. households and enterprises), and *H* (households, which may include "non-profit institutions in service of households"). Enterprises differ from households in that they do not consume.

On the income side, the shares of domestic institutions in factor incomes, *SHIF*, are defined on the basis of their stock (or endowment) shares (INS1); the stocks (*QFINS*) are defined

below in Section A.2.4. The factor incomes of domestic institutions, *YIF*, are a function of these shares, factor incomes (*YF*) net of direct taxes, and exogenous payments of factor incomes to the outside world (INS2). (*YF* was defined Section A.2.1.) Using this information, the total incomes of domestic non-government institutions, *YI*, are the sum of factor incomes, transfers from the government (indexed to the numéraire, in this case the CPI), transfers from abroad, and transfers from other institutions in *INSDNG* (INS3). (Government incomes are defined in a separate equation.)

The values for consumption and transfer spending by domestic non-government institutions are defined after deducting payments for direct taxes and savings. The mathematical statement treats direct tax rates as exogenous (policy-determined) but demonstrates alternative treatments for savings rates. The marginal propensity to save, *MPS*, is the product of an institution-specific rate (which may change over time) and a scaling parameter (INS4). If the latter is flexible, then total savings are adjusted endogenously in the context of restrictions on the total quantity or value of private investment financed by domestic non-government institutions. Here, the scaling parameter is fixed, meaning that investment spending must be flexible. (This is discussed in Section A.2.4.) Institution-specific savings values, *SAV*, are a linear function of *MPS* and income net of direct taxes, with an optional (non-zero) intercept, which is indexed to the numéraire (INS5). The presence of an intercept is essential when base-year data indicate that some household groups have negative savings – without a separate (and negative) intercept and the related assumption that marginal and average savings rates differ, higher incomes would in this setting reduce savings further below zero. Transfers from institutions in *INSDNG* to other institutions (in *INS*), *TRII*, are fixed shares of their incomes net of direct taxes and savings (INS6). For households, consumption spending, *EH*, is defined as income net of direct taxes, savings, and transfers to other institutions (INS7). Household consumption demands, *QH*, are a function of population, prices, and total spending (EH) (INS8); the demand functions are derived from the maximization of a Stone-Geary utility function subject to the total spending and prices. It is referred to as a linear expenditure system (LES) since spending on any commodity (the product of price and quantity) is a linear function of *EH* – this is evident if one multiplies both sides of INS8 by the price variable (*PQD*).

The remaining equations in this block define current government receipts and spending. Government receipts, *YG*, are the sum of tax revenues, domestic and foreign transfers, and factor incomes (INS9). Domestic transfers are exogenous and indexed to the numéraire; foreign transfers are exogenous in FCU. The taxes are made up of direct taxes on institutions and factors; domestic indirect taxes on sales, value-added, and activity revenues; export taxes; and import tariffs. To make the mathematical statement more easily digestible, value-added taxes (VATs) (*YTAXVAT*) are defined in a separate equation (INS10). Whether a given tax is part of an application depends on the database. As indicated, the VAT rates are disaggregated by commodity demanded, demander, and time period.

Current government spending, *EG*, is the sum of spending on consumption, domestic transfers, transfers abroad, and subsidies (INS11). The quantities of government consumption, *QG*, are defined on the basis of a trend term (*qgb*) that may be scaled selectively (by commodity and time period) (INS12); the impact of a given value for the scaling variable *QGSCAL* depends on the level of the parameter *qg01* – as indicated by its name, we propose that it be set at values between 0 and 1. In the current mathematical statement, *QGSCAL* is exogenous; if it were endogenous, it could be used to clear the government budget. Subsidy spending, *SUBCT*, is also defined in a separate equation (INS13); subsidy rates are similar to VAT rates in that they are disaggregated by commodity, demander, and time period.

Table A.2.3. Equations for current payments by domestic institutions

INS-1	$SHIF_{i,f,t} = \dfrac{QFINS_{i,f,t}}{\sum_{i' \in INSD} QFINS_{i',f,t}}$	$i \in INSD$ $f \in FVA$ $t \in T$	Shares of factor incomes to domestic institutions
INS-2	$YIF_{i,f,t} = SHIF_{i,f,t}\left(\left(1 - tf_{f,t}\right)YF_{f,t} - EXR_t \cdot trnsfr_{row,f,t}\right)$	$i \in INSD$ $f \in FVA$ $t \in T$	Factor income to domestic institutions
INS-3	$YI_{i,t} = \sum_{f \in FVA} YIF_{i,f,t} + trnsfr_{i,gov,t} \cdot \overline{CPI}_t$ $+ trnsfr_{i,row,t} \cdot EXR_t + \sum_{i' \in INSDNG} TRII_{i,i',t}$	$i \in INSDNG$ $t \in T$	Non-gov't institution income
INS-4	$MPS_{i,t} = mpsb_{i,t} \cdot \overline{MPSSCAL}_t$	$i \in INSDNG$ $t \in T$	Marginal propensity to save

INS-5	$SAV_{i,t} = \alpha_{i,t}^{sav} \cdot \overline{CPI} + MPS_{i,t}\left(1-ty_{i,t}\right)YI_{i,t}$	$i \in INSDNG$ $t \in T$	Non-gov't institution savings
INS-6	$TRII_{i,i'} = shii_{i,i'}\left(\left(1-ty_{i',t}\right)YI_{i',t} - SAV_{i',t}\right)$	$i \in INS$ $i' \in INSDNG$ $t \in T$	Institutional transfers
INS-7	$EH_{h,t} = \left(1-ty_{h,t}\right)YI_{h,t} - SAV_{h,t} - \sum_{i \in INS} TRII_{i,h,t}$	$h \in H$ $t \in T$	Household consumption expenditure
INS-8	$QH_{c,h,t} = pop_{h,t}\left(\gamma_{c,h,t}^{min} + \frac{\beta_{c,h}\left(\frac{EH_{h,t}}{pop_{h,t}} - \sum_{c' \in C} PQD_{c',h,t} \cdot \gamma_{c',h,t}^{min}\right)}{PQD_{c,h,t}}\right)$	$c \in C$ $h \in H$ $t \in T$	Household consumption demand
INS-9	$YG_t = \sum_{i \in INSDNG} ty_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t}$ $+\sum_{c \in C} tq_{c,t} \cdot PQS_{c,t} \cdot QQ_{c,t} + YTAXVAT_t$ $+\sum_{a \in A} ta_{a,t} \cdot PA_{a,t} \cdot QA_{a,t}$ $+\sum_{c \in C} te_{c,t} \cdot pwe_{c,t} \cdot QE_{c,t} \cdot EXR_t$ $+\sum_{c \in C} tm_{c,t} \cdot pwm_{c,t} \cdot QM_{c,t} \cdot EXR_t$ $+trnsfr_{gov,row,t} \cdot EXR_t$ $+\sum_{i \in INSDNG} TRII_{gov,i,t} + \sum_{f \in F} YIF_{gov,f,t}$	$t \in T$	Government current receipts
INS-10	$YTAXVAT_t =$ $\sum_{c \in C}\sum_{a \in A}\left(1-sub_{c,a,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,a,t} \cdot QINT_{c,a,t}$ $+\sum_{c \in C}\sum_{h \in H}\left(1-sub_{c,h,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,a,t} \cdot QH_{c,h,t}$ $+\sum_{c \in C}\left(1-sub_{c,gov,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,gov,t} \cdot QG_{c,t}$ $+\sum_{c \in C}\sum_{f \in FCAP}\left(\left(1-sub_{c,f,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,f,t}\right.$ $\left.\cdot capcomp_{c,f}\sum_{i \in INS} DKINS_{i,f,t}\right)$ $+\sum_{c \in C}\sum_{i \in INS}\left(1-sub_{c,dstk,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,dstk,t} \cdot qdstk_{c,i,t}$ $+\sum_{c \in C}\sum_{c' \in CDIS}\left(1-sub_{c,tacm,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,tacm,t} \cdot icm_{c,c',r} \cdot QMR_{c',r,t}$ $+\sum_{c \in C}\sum_{c' \in CDIS}\left(1-sub_{c,tace,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,tace,t} \cdot ice_{c,c',r} \cdot QER_{c',r,t}$ $+\sum_{c \in C}\sum_{c' \in C}\left(1-sub_{c,tacd,t}\right) \cdot PQS_{c,t} \cdot \left(1+tq_{c,t}\right) \cdot tva_{c,tacd,t} \cdot icd_{c,c'} \cdot QD_{c',r,t}$	$t \in T$	VAT revenue
INS-11	$EG_t = \sum_{c \in C} PQD_{c,gov,t} \cdot QG_{c,t} + \sum_{i \in INSDNG} trnsfr_{i,gov,t} \cdot \overline{CPI}_t$ $+trnsfr_{row,gov,t} \cdot EXR_t + SUBCT_t$	$t \in T$	Government expenditure

INS-12	$QG_{c,t} = qgb_{c,t}\left(1+qg01_{c,t}\cdot\overline{QGSCAL_t}\right)$	$c\in C$ $t\in T$	Government consumption
INS-13	$SUBCT_t =$ $\sum_{c\in C}\sum_{a\in A} sub_{c,a,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QINT_{c,a,t}$ $+\sum_{c\in C}\sum_{h\in H} sub_{c,h,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QH_{c,h,t}$ $+\sum_{c\in C} sub_{c,gov,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot QG_{c,t}$ $+\sum_{c\in C}\sum_{f\in FCAP} sub_{c,f,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot capcomp_{c,f}\sum_{i\in INS} DKINS_{i,f,t}$ $+\sum_{c\in C}\sum_{i\in INS} sub_{c,dstk,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot qdstk_{c,i,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS} sub_{c,tacm,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot icm_{c,c',r}\cdot QMR_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS} sub_{c,tace,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot ice_{c,c',r}\cdot QER_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in C} sub_{c,tacd,t}\cdot PQS_{c,t}\cdot\left(1+tq_{c,t}\right)\cdot icd_{c,c'}\cdot QD_{c',r,t}$	$t\in T$	Commodity subsidy

A.2.4. Investment, system constraints, and numéraire

This block covers investment spending by different institutions, how it is financed, and how the new capital that is generated feeds into the economy. The specification of investment and its financing makes it possible to specify the remaining system constraints, the markets for private capital factors, commodity balances, and the balance of payments.[14] (In Sections A.2.1 and A.2.3, we covered the markets for other factors and the government balance, respectively.) In addition, we here specify the numéraire, needed for CGE models like GEM-Core.

For the government, investment spending (or gross capital formation), *INVG*, is defined as the sum of government savings (the difference between current receipts and spending), domestic net financing (indexed to the numéraire), and foreign net financing (exogenous in FCU) (INV1). The sum of the two financing terms is the government primary deficit. These two terms are referred to as net financing items since they represent the difference between new borrowing and interest payments; the latter do not appear explicitly in the

[14] The term system constraints refers to constraints that are not perceived by individual actors (like household budget constraints) but which the economic system nevertheless most respect (like a labor market constraint which says that the quantity employed equals the labor force net of unemployment).

model.[15] For domestic non-government institutions, the corresponding variable, *INV*, is the sum of own savings and net financing from abroad minus claims on investment funding to finance the government and add to foreign reserves (INV2). Before translating investment spending into quantities of new capital, it is necessary to specify prices; in equation INV3, the unit prices of new capital stocks, *PK*, are defined as the product of the price of commodity *c* and the matrix of capital composition coefficients (which shows the quantities of commodities *c* used as inputs per unit of any new capital stock *f*) summed over all *c*.

The next three equations define investment quantities by destination (by type of capital stock) by government and non-government institutions and investment quantities by source (the use of commodity inputs in the production of new capital). For the government, the quantity of new capital stock *f*, $DKINS_{gov,f,t}$, is defined as investment spending net of spending on new inventory (gross fixed capital formation or GFCF), multiplied by the spending share for *f*, and divided by *PK* to transform into stock quantities (INV4). In the corresponding equation for non-government investment (INV5), GFCF is the sum of (a) investment net of stock change spending for institutions in INSDNG; and (b) foreign investment (the value of which is exogenous in FCU). GFCF is allocated across different capital stocks (if more than one) in fixed spending shares and transformed into quantities of new capital by dividing by *PK*.[16] Final investment demands (i.e. investment quantities defined by the source of inputs into the construction of new capital), *QINV*, are defined as the product of the capital composition matrix and investment by capital stock, summed over all capital stocks (INV6).

For any capital stock, the endowments held by domestic institutions (government and non-government), *QFINS*, are defined as the sum of (a) the stock held in the previous year net of depreciation; and (b) new investment in the previous year (INV7). For the stocks of other factors, endowments are defined exogenously (INV8). The values for endowments were

[15] Given simulation results for government and private net financing (domestic and foreign) and with the help of additional data on initial debt stocks, and real interest rates by time period, it is straightforward to compute debt stocks, which may be expressed in relation to other simulation results (like GDP or export values).

[16] From our experience, in most applications of models of this, it is preferable to have one type of private capital with constrained mobility. However, the option of multiple stocks may be useful if, for example, the model should mimic the allocation of foreign investment to specific sectors like mining.

used in Section A.2.1 to define the supply sides of markets for non-capital factors and, in Section A.2.3, to define the distributional shares for factor incomes.

The allocation of private capital stocks across activities responds to relative capital rents.[17] As an input to the formulation used, the average wage of private capital stock *f*, *WFAVG*, is defined as total rent to *f* divided by total employment of *f* (INV9). In equation INV10, the allocation of new private capital stock *f* to an activity *a*, *DKA*, is defined as the product of (a) an allocation based on current activity shares (i.e. total new investment in *f* times the current share of *a* in the use of *f*); and (b) an adjustment term that is above (below) unity if the wage of capital stock *f* in *a* is above (below) the economywide average, assuming a positive value for the parameter κ (Greek kappa; $\kappa \geq 0$). κ plays a crucial role in this formulation: the higher its value, the stronger the sensitivity of the allocation of new capital to differences in capital rents; if it is zero, the allocation of stock *f* does not change over time and if it is too high, capital rents may oscillate.[18]

Total employment of capital stock *f* in activity *a* in period *t*, $QF_{f,a,t}$, is defined as the stock installed in *t-1*, $QF_{f,a,t-1}$, net of depreciation, plus the quantity of new investment in stock *f* in *t-1* allocated to *a* , $DKA_{f,a,t-1}$ (INV11). This last equation may be seen as defining a set of activity-specific markets for capital stock *f* in which the quantity supplied (the right-hand side) is fixed within any period *t* (determined by past decisions) while the quantity demanded (the left-hand side) is determined by profit-maximization (cf. Section A.2.1). A

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[17] This approach that is presented may have been first developed in Dervis et al. (1982, pp. 175-178). Our treatment deviates in one respect: for simplicity, we use capital rents by activity instead of profit rates.

[18] With reference to INV10, by definition, $\sum_{a \in A} DKA_{f,a,t} = \sum_{i \in INSNG} DKINS_{f,i,t}$ for $f \in FCAPNG$ and $t \in T$. This can be shown as follows: For simplicity, replacing $\sum_{i \in INSNG} DKINS_{f,i,t}$ by DKI and $WF_{f,t} \cdot WFDIST_{f,a,t}$ by WFA_a, suppressing remaining *f* and *t* subscripts, and noting the definition of the average wage,

$$DKA_a = DKI \cdot \frac{QF_a}{\sum_{a' \in A} QF_{a'}} \cdot \left(1 + \kappa\left(\frac{WFA_a}{\overline{WF}} - 1\right)\right); \sum_{a \in A} DKA_a = DKI \cdot \sum_{a \in A}\left(\frac{QF_a}{\sum_{a' \in A} QF_{a'}} \cdot \left(1 + \kappa\left(\frac{WFA_a}{\overline{WF}} - 1\right)\right)\right)$$

$$= DKI\left(\frac{\sum_{a \in A} QF_a}{\sum_{a' \in A} QF_{a'}} + \kappa \cdot \sum_{a \in A}\left(\frac{WFA_a}{\overline{WF}} \frac{QF_a}{\sum_{a' \in A} QF_{a'}}\right) - \frac{\sum_{a \in A} QF_a}{\sum_{a' \in A} QF_{a'}}\right) = DKI\left(1 + \kappa\left(\frac{\overline{WF}}{\overline{WF}} - 1\right)\right) = DKI$$

wage variable defined over *f* and *a* is needed to clear this market. Accordingly, among the two wage variables that apply to any factor ($WF_{f,t}$ *and* $WFDIST_{f,a,t}$), equation INV12 fixes the economywide variable *WF* while leaving the activity-specific variable *WFDIST* flexible. The simulated values for the product of the two variables show the scarcity value of private capital stocks by activity. In sum, for private capital, it is assumed that installed stocks cannot be reallocated while the analyst controls the extent to which the allocation of new capital will shift toward sectors with relatively high capital rents.

For each domestic commodity, the demand side is now complete. The equation INV13 defines total composite demand for any commodity, *QQ*, as the sum of consumption, investment (fixed capital formation and stock changes), intermediate demands, and demands for trade and transportation services (due to domestic and foreign trade). As specified in Section A.2.2, these demands generate demands for domestic output and/or imports. The markets for domestic output sold domestically are cleared by the linked variables *PDD* and *PDS*; to exemplify, for the case of excess demand, increases in both price variables would simultaneously reduce domestic demands for domestic outputs and increase the quantities of output sold domestically (raising the total output level by raising profitability and raising the share of output sold domestically).

The statement of investment financing completes the flows in the balance of payments, which is expressed in FCU. Equations INV14 and INV15 state the current and capital accounts, respectively, with foreign savings, *SAVF*, as the linking variable. In the current account balance, inflows are due to exports, and transfers from abroad while outflows are caused by imports, transfers from domestic non-government institutions, and factor incomes. The variable *SAVF* measures the current-account deficit; if outflows (the right-hand side) are larger (smaller) than inflows (the left-hand side), foreign savings are positive (negative).[19] In the capital account balance, the current account deficit is financed by net foreign financing to government and non-government institutions and foreign investment, net of increases of foreign reserves. By influencing export and import quantities in opposite directions, raising or reducing the trade balance in FCU, adjustments in the exchange rate,

[19] The variable *SAVF* deviates from the definition of foreign savings given that interest payments are an implicit part of the capital account instead of being part of the current account. The variable *SAVF* could more accurately be referred to as the primary deficit of the nation.

EXR, clear the balance of payments, making sure that the level of foreign savings matches the level that is financed on the right-hand side of the capital account.

As a manifestation of Walras' law, in a CGE model like the one presented above, one equation should be removed to assure equality between the number of variables and independent equations; it is possible to check that the omitted equation holds in a post-calculation. Here we opt for the alternative of instead inserting one variable, named WALRAS, into one equation. Hence, the presence of *WALRAS* in the capital account of the balance of payments. In the absence of errors, the solution value for *WALRAS* should be (very close to) zero.

Finally, a well-specified CGE model like GEM-Core is homogeneous of degree zero in prices, meaning that only relative prices matter and that, if one set of relative prices solves the model, then any multiple of this set of prices would also solve the model (scaling all domestic prices and payments) without any influence on quantities. To anchor the price level, a price or price index, referred to as the numéraire, needs to be fixed, with the consequence that all other prices are measured relative to this numéraire. In this mathematical statement, the consumer price index, *CPI*, is the numéraire. Here, equation INV16 defines the CPI, which is fixed, on the basis of the base-year weights of household consumption payments by commodity and household type in total household consumption.[20]

Table A.2.4. Equations for investment, system constraints, and numéraire

INV-1	$INVG_t = \left(YG_t - EG_t\right) + ndfg_t \cdot \overline{CPI}_t + nff_{gov,t} \cdot EXR_t$	$t \in T$	Gov't primary deficit, investment value, and financing
INV-2	$INV_{i,t} = SAV_{i,t} + nff_{i,t} \cdot EXR_t - \left(\frac{SAV_{i,t}}{\sum_{i' \in INSDNG} SAV_{i',t}} \right) \left(ndfg_t \cdot \overline{CPI}_t + drf_t \cdot EXR_t \right)$	$i \in INSDNG$ $t \in T$	Non-gov't investment value and its financing

[20] As an alternative to CPI, the domestic producer price index (DPI) may serve as numéraire. In addition, it is often used as the denominator in the definition of the price-level-deflated (PLD) real exchange rate (REXR). Algebraically, with time subscripts omitted, $DPI = \sum_{c \in C} PDS_c \cdot dwts_c$ and $REXR = EXR / DPI$.

INV-3	$PK_{f,t} = \sum_{c \in C} PQD_{c,f,t} \cdot capcomp_{c,f}$	$f \in FCAP$ $t \in T$	Price of new capital
INV-4	$DKINS_{gov,f,t} = \frac{invshr_{f,gov,t} \cdot \left(INVG_t - \sum_{c \in C} PQD_{c,gov,t} \cdot qdstk_{c,gov,t} \right)}{PK_{f,t}}$	$f \in FCAPG$ $t \in T$	Gov't investment by government capital stock
INV-5	$DKINS_{i,f,t} = \frac{invshr_{f,i,t}}{PK_{f,t}} \cdot \left(\left(INV_{i,t} - \sum_{c \in C} PQD_{c,i,t} \cdot qdstk_{c,i,t} \right)\Big\|_{i \in INSDNG} + \left(invf_{i,t} \cdot EXR_t \right)\Big\|_{i \in INSROW} \right)$	$f \in FCAPNG$ $i \in INSNG$ $t \in T$	Non-gov't investment by private capital stock
INV-6	$QINV_{c,t} = \sum_{i \in INS} \sum_{f \in FCAP} capcomp_{c,f} \cdot DKINS_{i,f,t}$	$c \in C$ $t \in T$	Real investment demand (by source)
INV-7	$QFINS_{i,f,t} = QFINS_{i,f,t-1} \left(1 - depr_{f,t-1} \right) + DKINS_{i,f,t-1}$	$i \in INSD$ $f \in FCAP$ $t \in T$ $t \notin TMIN$	Accumulation of capital by domestic institutions
INV-8	$QFINS_{i,f,t} = qfinsb_{i,f,t}$	$i \in INSD$ $f \in FOTH$ $t \in T$	Exogenous institutional endowments for other factors
INV-9	$WFAVG_{f,t} = \frac{\sum_{a \in A} WF_{f,a,t} \cdot WFDIST_{f,a,t} \cdot QF_{f,a,t}}{\sum_{a \in A} QF_{f,a,t}}$	$f \in FCAPNG$ $t \in T$	Average wage (rent) by private capital stock
INV-10	$DKA_{f,a,t} = \left(\sum_{i \in INSNG} DKINS_{f,i,t} \right) \left(\frac{QF_{f,a,t}}{\sum_{a' \in A} QF_{f,a',t}} \right) \cdot \left(1 + \kappa_f \left(\frac{WF_{f,t} \cdot WFDIST_{f,a,t}}{WFAVG_{f,t}} - 1 \right) \right)$	$f \in FCAPNG$ $a \in A$ $t \in T$	Allocation of new private capital by activity
INV-11	$QF_{f,a,t} = QF_{f,a,t-1} \left(1 - depr_{f,t-1} \right) + DKA_{f,t-1}$	$f \in FCAPNG$ $a \in A, t \in T$ $t \notin TMIN$	Accumulation of private capital by activity
INV-12	$WF_{f,t} = wfb_f$	$f \in FCAPNG$	Exogenous economy-wide wage term for private capital
INV-13	$QQ_{c,t} = \sum_{h \in H} QH_{c,h,t} + QG_{c,t} + QINV_{c,t} + \sum_{i \in INSD} qdstk_{c,i,t} + \sum_{a \in A} QINT_{c,a,t} + QT_{c,t}$	$c \in C$ $t \in T$	Commodity balance

INV-14	$\sum_{c \in C} pwe_{c,t} \cdot QE_{c,t} + \sum_{ac \in INSD \cup F} trnsfr_{ac,row,t} + SAVF_t = \sum_{c \in C} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{i \in INSDNG} TRII_{row,i,t}}{EXR_t} + \sum_{f \in F} trnsfr_{row,f,t}$	$t \in T$	Current account of balance of payments
INV-15	$SAVF_t = \sum_{i \in INSD} nff_{i,t} + invf_t - drf_t + WALRAS_t$	$t \in T$	Capital account of balance of payments
INV-16	$\sum_{c \in C} \sum_{h \in H} PQD_{c,h,t} \cdot cwts_{c,h} = \overline{CPI_t}$	$t \in T$	Consumer price index

Appendix B: A User-Friendly Interface for GEM-Core[21]

The skills required to make productive use of CGE models in policy analysis are considerable, inter alia including strength in economics, modeling, and a variety of software. The purpose behind the development of a user-friendly interface for GEM-Core – and other models in the GEM Suite -- was to reduce the skills required in terms of modeling and software, permitting the analyst to focus on policy and economics, thereby making GEM-Core-based policy analysis more cost-effective.[22] The analyst who uses the interface, named ISIM, works exclusively in an Excel environment and receives substantial guidance throughout the analytical process. Knowledge of GAMS (the software in which GEM-Core is coded) and an editor (GAMS-IDE or other) is no longer needed.

This section briefly describes the structure of ISIM-MAMS, how users at different skill levels may use and interact with it, and the major steps involved in a typical application with a pre-defined data set. It is aimed at practitioners interested in applying the MAMS framework or developing similar interfaces for other models.

B.1. Overview of the interface

ISIM was developed using Visual Basic as an add-in for Excel 2007, later updated to also work with Excel 2010/2013/2016. The user is required to work in a current Windows environment and to have Excel 2007 or newer and GAMS installed on the computer.[23] The reason for using Excel as the front-end is that the Excel environment is familiar for most analysts, in effect removing the initial barrier for users to start working with ISIM.

ISIM is packaged with a set of models – including GEM-Core -- and data sets defined by country and year. These data sets have been developed by a core team drawing on the databases of existing GAMS-GEM applications (i.e., applications for models in the GEM Suite

[21] This appendix borrows from Lofgren et al. (2013).

[22] ISIM was originally developed as ISIM-MAMS, an interface for MAMS (see Lofgren et al. (2013)). Reducing the skills required to develop and apply economic models was also a major reason behind the initial development of GAMS (Bussieck and Meeraus, 2003, p. 138). The RunGTAP software for the GTAP model was developed with the same objective (Pearson and Horridge, 2005).

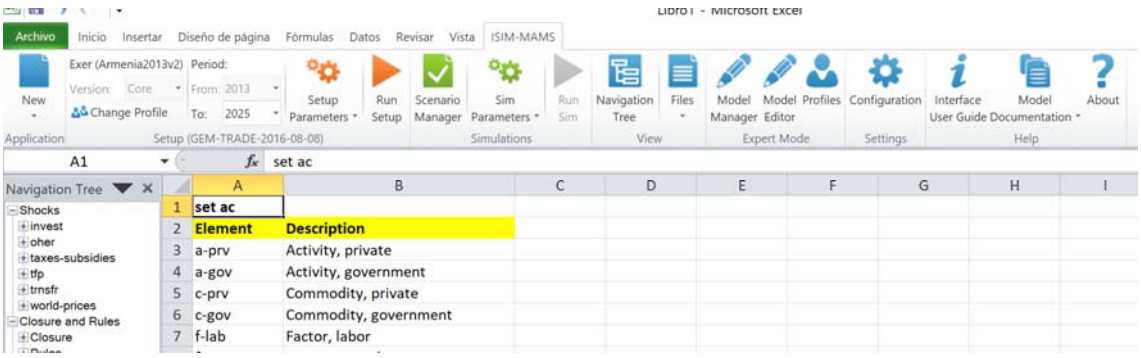
[23] One drawback of relying on Excel is that reprogramming of parts of ISIM was required to maintain compatibility across different Excel versions.

run using GAMS and Excel without the interface). Most users of ISIM are simply expected to do policy analysis using one or more applications, each of which is associated with an existing data set and includes a pre-programmed reference scenario. However, advanced users may also develop a database and a reference scenario working in GAMS-GEM and, after reading in the database as a new data set for ISIM (using its *Expert Mode*), shift to ISIM for policy analysis, perhaps collaborating with a broader group of analysts. The aim is that every user, irrespective of background, finds it more convenient to carry out the policy analysis step in ISIM rather than using the traditional GAMS-GEM alternative.

Assuming that GAMS and Excel 2007 or newer are already installed on the user's computer, the first step is to install ISIM, which comes in the form of an *.exe* file that, when executed, runs a standard installation routine. After installation, Excel has a new tab called ISIM that, when selected, opens an intuitive interface ribbon with user-friendly buttons. To run ISIM for an application (once defined), the user simply has to click on the *Run* button on the ISIM ribbon. The ISIM interface is connected to a database that, for each data set, stores definitions of sets (including commodities, activities, factors, and institutions) and parameters (including default elasticities, closure and rules/policies) that are used to define scenarios. The rest of the relevant country data set (other than the application database) is stored in the ISIM installation folder, mostly in GDX and Excel files.

Assuming that the user is satisfied to work with one of the existing data sets, the next steps are to: (1) open Excel 2007 or newer; (2) select the ISIM tab (see Figure B.1); (3) create a new application and associate it with one of the available data sets; (4) run and optionally modify the pre-defined reference scenario; (5) define and run additional scenarios such as "base" (that may be identical to the reference scenario) and, most likely, other scenarios of interest; and (6) access the results inside the same Excel file, presented in tables and graphs. Throughout the process, parameters and other items are hyperlinked to relevant segments in the GEM-Core User Guide (Lofgren and Cicowiez, 2017b), which is included with ISIM. Each application resides in an Excel file (named by the user) that can be used by others who have ISIM installed.

Figure B.1. The ISIM Ribbon and Navigation Tree

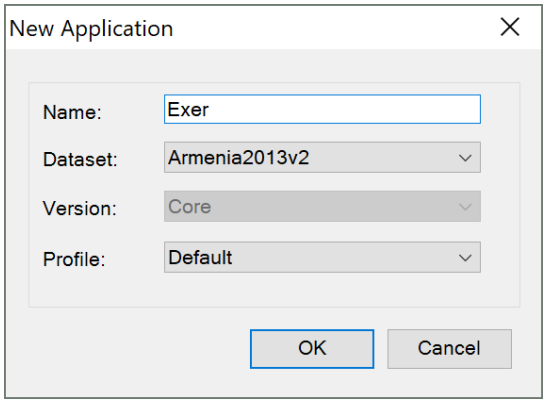


B.2. Using ISIM

Selection of database and model version

This section shows how to create an ISIM application named "example" based on the "Armenia2013v2" data set and using the GEM-Core model (see Figure B.2).[24] ISIM will load into Excel the data needed to define scenarios for the selected application, including the elements in the sets that are used to define shocks (e.g., the elements in the set of exported commodities are loaded to define world export prices; or the elements in institution sets so as to define inter-institutional transfers).

Figure B.2. New Application dialog box



ISIM permits changes in and creation of application databases; the user can change selected elements of these data sets, like elasticities, closures and rules. To facilitate the navigation

[24] As explained in Section 4, two versions of MAMS exist: Core (a standard, dynamic-recursive CGE model), and MDG (an extended dynamic-recursive CGE model designed for MDG and human development analysis).

across the different sections of the Excel file, the user is provided with a button for the *Navigation Tree*, where the analyst can click on the element of interest (see Figure B.1).

In addition, (advanced) users can add new country data sets to ISIM and change other aspects of the GEM-Core program by editing relevant files in the ISIM\GEM-Core installation folder, using Excel and a text editor.

The reference scenario

The first step in performing counterfactual simulations with ISIM is the construction of a (dynamic) baseline scenario. To help the user carry out this task, ISIM includes, for each country data set, a pre-defined reference scenario. Key parameters of this scenario can be changed inside ISIM, including the GDP growth rate, all model elasticities (including those related to trade, household expenditure, the reservation wage, and the impact of share of trade in real GDP on total factor productivity), as well as closures and other rules, the latter covering the government budget, the balance of payments, the savings-investment balance, factor markets, and various payments, split into government and non-government depending on whether the government is involved or not. The rule chosen for any payment is overwritten if, according to the related closure setting, the payment in question is a free variable; for example, the specification that direct taxes are determined on the basis of exogenous tax rates is overwritten if, according to the government closure rules, changes in direct taxes clear the government budget. In addition, the user can configure the ISIM Poverty Module (see Lofgren and Cicowiez (2017b)). Specifically, the parameters of the Poverty Module allow the user to change: (1) the approach to compute poverty; (2) the welfare index (income/consumption); (3) the initial poverty rate; and (4) the growth elasticity of poverty.

To exemplify, Figures B.3a and B.3b demonstrate how the analyst can change the elasticities of substitution between primary factors of production; the default values can always be restored. Once defined, the reference scenario can be run by clicking the *Run Setup* button; ISIM will automatically call GAMS in order to run GEM-Core.

Figure B.3.a. Changing the elasticity of substitution between factors of production

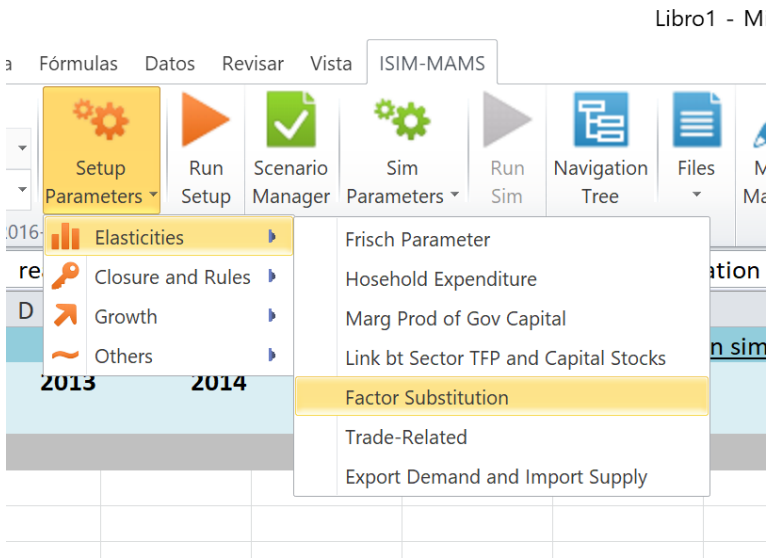
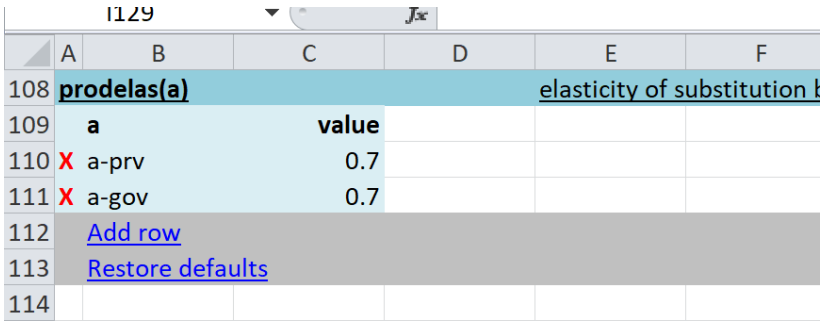


Figure B.3.b: Changing the elasticity of substitution between factors of production – cont.



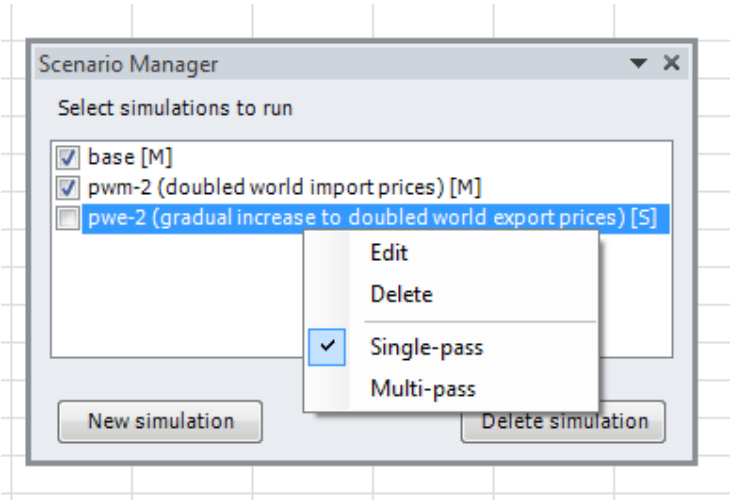
Defining and running scenarios for analysis

By default, ISIM generates a scenario called *base*. The user can define additional scenarios and introduce policy changes and exogenous shocks, including changes in world prices of exports and imports, foreign aid, taxation, public spending (and its allocation in other models in the GEM Suite). All the defined counterfactual scenarios are saved in the ISIM application-specific Excel file. The types of changes that can be introduced relative to the reference scenario reflect what seemed relevant to include in light of experience from a large number of CGE applications. The interface validates the input from the user in order to reduce the likelihood that simulations will fail to run without error.

To create a new simulation or delete an existing one, the user clicks on the *Scenario Manager* button, opening the window shown in Figure B.4. By clicking or unclicking the box

in front of each scenario, the user decides which simulations to run (there is no need to run all the defined scenarios). By right-clicking above any scenario, the user can edit the name and the explanatory text, delete, clone, and change the solution order of the simulation. In the example, the pwm-2 simulation will be run, while the pwe-2 will not. The elements under Shocks and Closure and Rules in the *Navigation Tree* show what can be changed in the definition of the non-reference scenarios. If no changes are made for a scenario, then it is identical to the reference scenario. In order for the base scenario to function as the benchmark to which other scenarios are compared, it may be preferable to leave it unchanged (i.e., identical to the reference scenario).

Figure B.4: The Scenario Manager



More specifically, the user can make changes in a set of items, grouped into the following categories:

- external shocks: changes in (1) the world price of exports and imports; (2) foreign direct investment; and (3) foreign borrowing.
- total factor productivity shocks: changes in (1) total factor productivity, by activity and time period; and (2) real GDP at factor cost.
- demographic shocks: changes in (1) population size, by representative household or other population segment; and (2) changes in the growth rate of non-capital factors.
- transfers shocks: changes in (1) transfers to non-household institutions or factors; and (2) per-capita transfers to households.

- closures and rules: changes in closures and rules – similar to the setting of closures and rules for the pre-programmed reference scenario. Among other things, the rules section allows defining scenarios with changes in (1) government spending and receipts, and (2) transfers between institutions

As an example, Figure B.5 shows how to define a 50 percent increase in the world price of agricultural exports during 2014-2030 using an application based on the "Armenia2013v2" data set; the other shocks can be similarly defined.

Figure B.5. Defining a shock to the world price of exports

D16 fx

	A	B	C	D	E	F	G	H	I
1		**pwesim(sim,c,t)**		export price for c (foreign currency) in simulation sim (deviation wrt base)					
2		**sim**	**c**	**2013**	**2014**	**2015**	**2016**	**2017**	**201**
3	X	pwe-2	c-prv	1	1.5	1.5	1.5	1.5	1
4		Add row							
5									
6									

Review and further processing of simulation results

Once the selected scenarios have been run with success, results can be accessed via the ISIM interface by clicking on *Reports* via the *Navigation Tree*. In addition to providing simulated variable results as levels, growth rates, and GDP shares – all accessible through a pivot table inside Excel as well as GAMS GDX files – the interface generates pre-defined tables and figures (see Figure B.6 for an example of a pre-defined table). By clicking on the *Configuration* button, the user can select which pre-programmed tables are generated, start and end years for these tables, the order in which the result tables will appear, and whether or not to generate a pivot table and chart with raw model results. Irrespective of the settings under *Configuration* | *Reports*, the user can access all simulation results using the report GDX files through the *View* | *Files* menu option.

Figure B.6. GEM-Core results

J11

	A	B	C	D	E	F
	Macro Report Contents					
	real macro indicators growth (% from first to final year)					
		baseyr	**base**	**inf-tdir**	**inf-fbor**	**inf-dbor**
	Absorption	5,459,383.10	3.51	4.33	4.49	4.14
	PrvCon	3,971,174.60	3.50	3.98	4.12	3.89
	FixInv	966,365.30	3.87	6.26	6.53	5.68
	StockChange	-21,875.20	3.41	3.41	3.41	3.41
	GovCon	543,718.40	2.92	2.92	2.92	2.92
	Exports	1,045,926.59	3.40	5.07	5.15	4.71
0	**Imports**	1,949,671.49	3.70	4.50	4.68	4.31
1	**GDPMP**	4,555,638.20	3.40	4.44	4.57	4.21
2	**NetIndTax**	589,734.70	3.60	4.57	4.72	4.36
3	**GDPFC**	3,965,903.50	3.41	4.46	4.59	4.22
4	**REXR**	1.00	-0.37	0.00	-0.05	-0.04
5	**Wage**	1.00	2.27	3.10	3.24	2.85
6	**UnempRat**	16.18	9.56	8.34	8.09	8.75
7						

Input validation and error messages

Before calling GAMS to run a selected simulation, ISIM validates the definition of shocks, elasticities, and/or closures and rules. In case errors are found, messages appear in pop-up windows and the user will have to check the red Excel cells, located in the sheets whose label also turned to red. Besides, an error summary sheet shows the list of generated validation errors. In case the solution of ISIM ends with an error, the user is offered the chance to inspect the ISIM log file or the GAMS listing file. The log and listing file viewer allows the user to navigate through the ISIM and/or GAMS errors. The log file is intended for users with no GAMS knowledge. On the other hand, the GAMS listing file provides the raw GAMS results and error messages. In case GEM-Core --or any other model within ISIM -- is successfully solved, ISIM will add or update the report sheets (i.e., it will add extra worksheets in Excel).

Appendix C: Additional Simulation Results

Table C.1. Real macro indicators for base and non-base simulations (% annual growth 2018-2030)

Item	2017	base	ginv-tdir	ginv-dbor	ginv-fbor	pwe	remit
Absorption	105.6	4.98	5.11	4.75	5.27	5.21	5.16
Consumption, private	80.8	4.96	4.94	4.71	5.14	5.26	5.19
Investment	19.2	4.96	5.78	4.78	5.82	4.98	5.01
Investment, private	14.4	4.96	4.95	3.44	5.01	4.99	5.02
Investment, government	4.8	4.96	7.89	7.89	7.89	4.96	4.96
Consumption, government	11.7	5.09	5.09	5.09	5.09	5.09	5.09
Exports	19.8	4.97	5.14	4.70	4.91	5.32	4.70
Imports	25.3	4.97	5.10	4.76	5.46	5.85	5.38
GDP factor cost	92.5	4.98	5.12	4.74	5.14	5.05	5.00
Real exchange rate (index)	1.00	0.01	0.02	0.00	-0.16	-0.52	-0.20
Unemployment rate (%)	5.36	4.33	4.26	4.55	4.24	4.19	4.30

Note: Except for unemployment, the 2017 column shows levels as GDP shares (%), while the simulation columns show annual growth rates 2018-2030. For unemployment, the 2017 and simulation columns show the rates in 2017 and 2030, respectively.

Source: Authors' calculations based on simulation results.

Table C.2. Macro indicatores in 2017 and for simulations in 2030 (% of nominal GDP)

Item	2017	base	ginv-tdir	ginv-dbor	ginv-fbor	pwe	remit
Absorption	105.6	105.6	105.5	105.8	107.0	105.0	107.3
Consumption, private	80.8	80.8	79.2	80.9	80.7	81.0	82.6
Investment	19.2	19.2	20.9	19.4	20.9	18.6	19.2
Investment, private	14.4	14.4	14.1	12.3	14.2	14.0	14.4
Investment, government	4.8	4.8	6.7	7.1	6.7	4.6	4.8
Consumption, government	11.7	11.7	11.5	11.9	11.5	11.4	11.7
Exports	19.8	19.8	19.9	19.7	18.8	20.7	18.6
Imports	25.3	25.4	25.4	25.4	25.8	25.8	25.9
GDP factor cost	92.5	92.5	92.5	92.5	92.4	92.6	92.4
Foreign savings	4.9	4.9	4.8	5.0	6.3	4.4	4.7
Government savings	1.8	1.8	3.8	1.6	2.1	1.8	1.8
Domestic non-gov savings	6.4	6.4	6.3	6.4	6.4	6.4	6.6
Foreign government debt	31.0	33.9	33.3	34.9	49.2	30.9	32.9
Foreign private debt	5.4	6.9	6.8	7.1	6.6	6.3	6.7
Domestic government debt	17.9	26.0	25.5	47.7	25.4	25.1	25.8

Source: Authors' calculations based on simulation results.

Table C.3. Real sectoral value added for base and non-base simulations (% annual growth 2018-2030)

	2017	base	ginv-tdir	ginv-dbor	ginv-fbor	pwe	remit
Private	89	4.97	5.12	4.73	5.15	5.05	5.00
Government	3	5.09	5.09	5.09	5.09	5.09	5.09
Total	93	4.98	5.12	4.74	5.14	5.05	5.00

Note: The 2017 column shows levels as GDP shares (%), while the simulation columns show annual growth rates 2018-2030.

Source: Authors' calculations based on simulation results.

Table C.4. Government receipts and spending in 2017 and for simulations in 2030 (% of nominal GDP)

Item	2017	base	ginv-tdir	ginv-dbor	ginv-fbor	pwe	remit
Recurrent receipts							
Direct taxes	5.0	5.0	6.8	5.0	5.0	4.7	4.8
Social contributions	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Activity taxes	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Commodity taxes	5.7	5.7	5.7	5.7	5.7	5.6	5.7
Tariffs	1.4	1.4	1.4	1.5	1.5	1.5	1.5
Export taxes	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Domestic transfers	1.8	1.8	1.8	1.8	1.8	1.8	1.9
Foreign transfers	0.9	0.9	0.8	0.9	0.8	0.8	0.8
Total	15.1	15.1	16.9	15.2	15.2	14.8	15.1
Recurrent spending							
Consumption	11.7	11.7	11.5	11.9	11.5	11.4	11.7
Domestic transfers	1.6	1.6	1.5	1.6	1.5	1.5	1.6
Foreign transfers	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Total	13.4	13.4	13.1	13.6	13.1	13.0	13.3
Savings*	1.8	1.8	3.8	1.6	2.1	1.8	1.8
Investment	4.8	4.8	6.7	7.1	6.7	4.6	4.8
Surplus**	-3.0	-3.0	-3.0	-5.5	-4.6	-2.9	-3.0
Financing							
Net domestic financing	1.5	1.5	1.5	3.9	1.5	1.5	1.5
Net foreign financing	1.5	1.5	1.5	1.6	3.1	1.4	1.5
Total	3.0	3.0	3.0	5.5	4.6	2.9	3.0

*Savings is the difference between the totals for recurrent receipts and recurrent spending.

**Surplus is the difference between savings and investment.

Source: Authors' calculations based on simulation results.

Table C.5. Balance of payments in 2017 and for simulations in 2030 (% of nominal GDP)

Item	2017	base	ginv-tdir	ginv-dbor	ginv-fbor	pwe	remit
Current account, inflows of foreign exchange							
Exports	19.8	19.8	19.9	19.7	18.8	20.7	18.6
Transfers to non-government	2.8	2.8	2.7	2.9	2.7	2.5	4.6
Transfers to government	0.9	0.9	0.8	0.9	0.8	0.8	0.8
Factor income	0.3	0.3	0.3	0.3	0.2	0.2	0.2
Foreign savings	4.9	4.9	4.8	5.0	6.3	4.4	4.7
Total	28.5	28.5	28.5	28.7	28.9	28.7	29.0
Current account, outflows of foreign exchange							
Imports	25.3	25.4	25.4	25.4	25.8	25.8	25.9
Transfers from non-government	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Transfers from government	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Factor income	2.6	2.6	2.6	2.7	2.5	2.4	2.5
Total	28.5	28.5	28.5	28.7	28.9	28.7	29.0
Capital account							
Net foreign financing to non-government	0.4	0.4	0.4	0.4	0.3	0.3	0.4
Net foreign financing to government	1.5	1.5	1.5	1.6	3.1	1.4	1.5
Foreign direct investment	2.9	2.9	2.8	2.9	2.7	2.6	2.8
Change in foreign reserves	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Total	4.9	4.9	4.8	5.0	6.3	4.4	4.7

Source: Authors' calculations based on simulation results.