A Static Open Economy Computable General Equilibrium Model

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January 10, 2018

Introduction

In this document, we present the mathematical statement of a static open economy (SOE) computable general equilibrium (CGE) model with standard characteristics. The model is relatively simple, has standard characteristics, and may be used to assess (ex-ante) the effects of various economic shocks on an economy. The SOE model is a close relative of the IFPRI Standard Model (Lofgren et al., 2002). The model, coded in GAMS (General Algebraic Modeling System), can be run through ISIM, a user-friendly interface (Cicowiez et al., 2013).

Model Mathematical Statement

This appendix provides a detailed mathematical statement of the SOE model. The model equations are divided into four blocks:

- Production activities and factor markets;
- Domestic and foreign trade;
- Domestic institutions; and
- System constraints and price indices.

Tables A.1-A.5 explain notational principles and define model sets, variables, and parameters. Drawing on these tables and set of tables with the model equations (Tables A.6-A.9), we subsequently provide a detailed presentation of the model equations with one section for each block. In the mathematical presentation, the settings for closure rules (for government budget,

savings-investment payments, and factor and commodity markets) and other assumptions match what was used in the simulations of this paper.

Table A.1. Notational principles

Items	Notation	Example
Sets	Lower-case Latin letters as subscripts to variables and parameters	exemplified on the following rows
Endogenous variables	Upper-case Latin letters (without a bar)*	QG_c
Exogenous variables	Upper-case Latin letters with a bar*	$\overline{QFS}_f$
Parameters	Lower-case Latin letters* or lower-case Greek letters (with or without superscripts)	$ica_{c,a}$; ρ_c^q

*The names of Latin letter variables and parameters that refer to prices, quantities, and wages (rents) start with *P*, *Q*, and *WF*, respectively.

Table A.2. Sets

Name	Description
$a \in A$	activities (production sectors or industries)
$c \in C$	commodities (i.e., goods and services)
$c \in CD(\subset C)$	commodities with domestic sales of domestic output
$c \in CE(\subset C)$	exported commodities
$c \in CM(\subset C)$	imported commodities
$c \in CT(\subset C)$	transactions commodities (trade and transport services paid for under distribution margins)
$f \in F$	factors
$i \in INS$	institutions
$i \in INSD(\subset INS)$	domestic institutions
$i \in INSDNG(\subset INSD)$	domestic non-government institutions
$h \in H(\subset INSDNG)$	representative households (RHs)

Table A.3. Variables

Name	Description
CPI	consumer price index
DPI	domestic producer price index (PDS-based)
EG	total current government expenditure
EH_h	household consumption expenditure

EXR	exchange rate (local currency per unit of foreign currency
MPS_i	marginal propensity to save for domestic non-government institutions *i* (in *INSDNG*)
$MPSSCAL$	MPS_i scaling factor
PA_a	output price of activity *a*
PDD_c	demand price for commodity *c* produced and sold domestically
PDS_c	supply price for commodity *c* produced and sold domestically
PE_c	export price for *c* (domestic currency)
PM_c	import price for *c* (domestic currency)
PQ_c	composite commodity price for *c*
PX_c	producer price for commodity *c*
QA_a	level of activity *a*
QD_c	quantity sold domestically of domestic output *c*
QE_c	quantity of exports of commodity *c*
$QF_{f,a}$	quantity demanded of factor *f* by activity *a*
QFS_f	supply of factor *f* (in *FNPROX*)
QG_c	quantity of government consumption of commodity *c*
$QGSCAL$	government consumption scaling factor
$QH_{c,h}$	quantity consumed of commodity *c* by household *h*
$QINT_{c,a}$	quantity of commodity *c* as intermediate input to activity *a*
$QINV_c$	quantity of investment demand for commodity *c*
$QINVSCAL$	investment scaling factor
QM_c	quantity of imports of commodity *c*
QQ_c	quantity of composite supply and demand of commodity *c*
QT_c	quantity of transactions demand for commodity *c*
QX_c	quantity of domestic output of commodity *c*
$RSAVG$	real government savings (CPI-indexed)
$SAVF$	foreign savings (FCU)
$SAVG$	government savings
SAV	savings of domestic non-government institution *i* (in *INSDNG*)
$TRII_{i,i'}$	transfers to institution *i* (in *INS*) from domestic non-government institution *i'* (in *INSDNG*)
TY_i	rate of direct tax on domestic non-government institution *i* (in *INSDNG*)
$TYSCAL$	scaling variable for direct tax on domestic non-government institutions
$WALRAS$	variable checking satisfaction of Walras' law
WF_f	economy-wide wage of factor *f*
$WFDIST_f$	wage distortion factor for factor *f* in activity *a*
YF_f	income of factor *f*
YG	government current revenue
YI_i	income of (domestic non-government) institution *i* (in *INSDNG*)
$YIF_{i,f}$	income of institution *i* (in *INS*) from factor *f*

Table A.4. Latin letter parameters

Name	Description
$cwts_c$	weight of commodity *c* in the CPI
$dwts_c$	weight of commodity *c* in the DPI
$ica_{c,a}$	quantity of intermediate input c per unit of activity *a*
$icd_{c,c'}$	transactions input of *c* per unit of commodity *c'* produced and sold domestically
$ice_{c,c'}$	transactions input of *c* per unit of commodity *c'* that is exported
$icm_{c,c'}$	transactions input of *c* per unit of commodity *c'* that is imported
$mpsb_i$	base-year marginal propensity to save for domestic non-government institution *i* (in *INSDNG*)
pwe_c	export price for *c* (foreign currency)
pwm_c	import price for *c* (foreign currency)
$qdstk_c$	change in stock (inventories) of *c*
qgb_c	base-year quantity of government consumption of commodity *c*
$qinvb_c$	base-year quantity of investment (GFCF) demand for *c*
$shif_{i,f}$	share for institution *i* (in *INS*) in the income of factor *f*
$shii_{i',i}$	share of institution *i'* (in *INS*) in the disposable income of domestic non-government institution *i* (in *INSDNG*)
$trnsfr_{ac,i}$	transfers from institution i (government or rest of world) to institution *i* (in *INS*) or factor *f* (in *F*)
ta_a	rate of tax on gross output value for activity *a*
te_c	rate of export tax on commodity *c*
tf_f	rate of direct tax on factor *f*
tm_c	rate of import tariff on commodity *c*
tq_c	rate of sales tax on commodity *c*
tyb_i	base rate of direct tax on domestic non-government institution *i* (in *INSDNG*)

Table A.5. Greek letter parameters

Name	Description
$\beta_{c,h}$	share parameter in LES function for household consumption of commodity *c*
$\gamma_{c,h}$	subsistence quantity in LES function for household consumption of commodity *c*
δ_c^{dd}	share parameter for domestic purchases in Armington function for commodity *c*
δ_c^{ds}	share parameter for domestic sales in CET function for commodity *c*
δ_c^{e}	share parameter for exports in CET function for commodity *c*
δ_c^{m}	share parameter for imports in Armington function for commodity *c*
δ_c^{va}	share parameter for factor *f* in CES VA function of activity *a*
$\theta_{a,c}$	yield of output c per unit of activity *a*
ρ_c^{q}	exponent in Armington function for commodity *c*

ρ_a^{va}	exponent in CES VA function for activity *a*
ρ_c^{x}	exponent in CET function for commodity *c*
σ_c^{q}	elasticity of substitution between purchases of domestic output and imports in Armington function for *c*
σ_c^{va}	elasticity of substitution between factors in CES VA function of activity *a*
σ_c^{x}	elasticity of transformation between domestic sales and exports in CET function for *c*
φ_c^{q}	shift parameter in Armington function for commodity *c*
φ_c^{va}	shift parameter for CES VA function of activity *a*
φ_c^{x}	shift parameter in CET function for commodity *c*

Table A.6. Equations for production activities and factor markets

PRD1	$QA_a = \varphi_a^{va}\left(\sum_{f\in F} \delta_{f,a}^{va} \cdot QF_{f,a}^{-\rho_a^{va}}\right)^{\frac{-1}{\rho_a^{va}}}$	$a \in A$	Value-added
PRD2	$\overline{QF}_{f,a} = \left(\frac{PVA_a}{\overline{WF}_f \cdot WFDIST_{f,a}}\right)^{\sigma_a^{va}} \left(\delta_{f,a}^{va}\right)^{\sigma_a^{va}} (\varphi_a^{va})^{\sigma_a^{va}-1} \cdot QA_a$	$f \in FNLAB$ $a \in A$	Factor demands (non-labor)
PRD3	$QF_{f,a} = \left(\frac{PVA_a}{WF_f \cdot \overline{WFDIST}_{f,a}}\right)^{\sigma_a^{va}} \left(\delta_{f,a}^{va}\right)^{\sigma_a^{va}} (\varphi_a^{va})^{\sigma_a^{va}-1} \cdot QA_a$	$f \in FLAB$ $a \in A$	Factor demands (labor)
PRD4	$QINT_{c,a} = ica_{c,a} \cdot QA_a$	$c \in C$ $a \in A$	Intermediate demands
PRD5	$QX_c = \sum_{a\in A} \theta_{a,c} \cdot QA_a$	$c \in C$	Output
PRD6	$PVA_a = PA_a(1 - ta_a) - \sum_{c\in C} PQ_c \cdot ica_{c,a}$	$a \in A$	Value-added price
PRD7	$PA_a = \sum_{c\in C} \theta_{a,c} \cdot PX_c$	$a \in A$	Activity price
PRD8	$QFS_f = \sum_{a\in A} \overline{QF}_{f,a}$	$f \in F$	Factor markets
	$\overline{QFS}_f = \sum_{a\in A} QF_{f,a}$		
PRD9	$YF_f = \sum_{a\in A} \overline{WF}_f \cdot WFDIST_{f,a} \cdot \overline{QF}_{f,a} + trnsfr_{f,row} \cdot EXR$	$f \in FNLAB$	Factor income (non-labor)
PRD10	$YF_f = \sum_{a\in A} WF_f \cdot \overline{WFDIST}_{f,a} \cdot \overline{QF}_{f,a} + trnsfr_{f,row} \cdot EXR$	$f \in FLAB$	Factor income (labor)

Production activities and factor markets

The equations for this block are presented in Table A.6. Equations PRD1-PRD3 are the first-order conditions for the optimization problem solved by the representative firm in each industry or activity (i.e., cost minimization/profit maximization).[1] The value-added production technology is CES (Constant Elasticity of Substitution; equation PRD1). In the demand functions for factors *f*, the (unit) wage in activity *a* is computed as $WF_f \cdot WFDIST_{f,a}$, where $WFDIST_{f,a}$ is a "distortion" factor that allows modeling cases in which the factor remuneration differs across activities.[2] As discussed in Lofgren et al. (2002), this formulation facilitates implementing alternative closures (i.e., mechanisms to equalize quantities supplied and demanded) in the factor markets.[3] In this application, the factors in *FNLAB* are all activity-specific and $WFDIST_{f,a}$ is the variable that clears these activity-specific markets.

For the factor subset in *FNLAB* – non-labor factors that are sector-specific -- equation PRD2 provides the demand functions. Given that, for this subset, factor supplies are exogenous by activity (as indicated by the bar on the factor employment variable $QF_{f,a}$), their wage "distortion" variables, $WFDIST_{f,a}$, clear the markets. Equation PRD3 shows the demand functions for labor factors that are perfectly mobile across sectors. Equation PRD4 defines intermediate input demands using Leontief coefficients, which represent exogenous quantities of intermediate inputs by commodity and activity ($ica_{c,a}$).

Equation PRD5 computes the production of each commodity on the basis of the $\theta_{a,c}$ parameter, which is a fixed output coefficient (the production of *c* per unit of activity *a*). Thus, like the supply-and-use tables, our model differentiates between activities and commodities (or

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[1] To make it easier to read the equations, multiplication signs are inserted between multiplied items unless one or both are parenthesized.

[2] In this presentation we assume that its value is constant only for labor factors, the only factor in FLAB (see below).

[3] Besides, for the factors considered as specific, equation (PRD3) is interpreted as an equilibrium condition between factor supply and demand.

products): an activity may produce more than commodity and the same commodity may be produced by more than one activity.

Equations PRD6 and PRD7 define prices related to each production activity. In equation PRD6, the price of value added (the payment to value added per unit of each activity) is defined as the activity price (the payment received for outputs per unit of the activity net of the tax on activity revenue) minus the cost of intermediate inputs per unit of the activity. The latter cost is the product of the above-mentioned intermediate input coefficient ($ica_{a,c}$) and the price of each intermediate, summed over all intermediates. Equation PRD7 defines the activity price as the product of output yields per unit of the activity, $\theta_{a,c}$, and related producer output prices, summed over all outputs.

The remaining equations of this block are factor-related, imposing equality between quantities demanded and supplied. As noted, in the current application, all factors in the set *FNLAB* are activity-specific with the variable $WFDIST_{f,a}$ clearing their markets (cf. equation PRD2). Given this, the economy-wide market equilibrium condition, PRD8, becomes superfluous; here it simply defines total factor supply (which is endogenous) as the sum of the exogenous demands in the different activities. To facilitate a switch to closures where this condition matters, this equation is nevertheless retained for all factors.[4] On the other hand, for factors in the set *FLAB* equation PRD8 is the economy-wide market equilibrium condition.

In equations PRD9 and PRD10, the total income of each factor *f*, YF_f, is defined for sector-specific and mobile factors, respectively. In both equations, the first term on the right hand side corresponds to the total factor payments from activities while the second term defines transfers to factors from the rest of the world.[5]

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[4] Different treatments are possible. For example, total supply (employment) may be exogenous while mobility may be free across all activities; for such factors, QFS_f and $WFDIST_{f,a}$ would be exogenous while QF_f and WF_f would be endogenous, leaving the total number of endogenous variables unchanged. In this case, equation PRD2 is an essential part of the model.

[5] Note that the $trnsfr_{f,row}$ parameter is expressed in foreign currency units.

Table A.7. Equations for domestic and foreign trade

TRD1	$PE_c = (1 - te_c)EXR \cdot pwe_c - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c',c}$	$c \in C$	Export price
TRD2	$PM_c = (1 + tm_c)EXR \cdot pwm_c + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c',c}$	$c \in C$	Import price
TRD3	$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c',c}$	$c \in C$	Domestic demander price
TRD4	$QQ_c = \varphi_c^q \left(\delta_c^m \cdot QM_c^{-\rho_c^q} + \delta_c^{dd} \cdot QD_c^{-\rho_c^q}\right)^{\frac{-1}{\rho_c^q}}$	$c \in CM \cap CD$	Composite supply (tradables)
TRD5	$QQ_c = QM_c + QD_c$	$(c \in CM \cap c \notin CD) \cup (c \in CD \cap c \notin CM)$	Composite supply (non-tradables)
TRD6	$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c} \frac{\delta_c^m}{\delta_c^{dd}}\right)^{\frac{1}{1+\rho_c^q}}$	$c \in CM \cap CD$	Import-domestic demand ratio
TRD7	$PQ_c \cdot QQ_c = (PM_c \cdot QM_c + PDD_c \cdot QD_c)(1 + tq_c)$	$c \in C$	Composite price
TRD8	$QX_c = \varphi_c^x \left(\delta_c^e \cdot QE_c^{\rho_c^x} + \delta_c^{ds} \cdot QD_c^{-\rho_c^x}\right)^{\frac{-1}{\rho_c^x}}$	$c \in CE \cap CD$	Output transfor-mation (tradables)
TRD9	$QX_c = QE_c + QD_c$	$(c \in CE \cap c \notin CD) \cup (c \in CD \cap c \notin CE)$	Output transfor-mation (non-tradables)
TRD10	$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \frac{\delta_c^{ds}}{\delta_c^e}\right)^{\frac{1}{\rho_c^x - 1}}$	$c \in CE \cap CD$	Export-domestic demand ratio
TRD11	$PX_c \cdot QX_c = (PE_c \cdot QE_c + PDS_c \cdot QD_c)$	$c \in C$	Producer output price
TRD12	$QT_c = \sum_{c' \in C} icd_{c,c'} \cdot QD_{c'} + \sum_{c' \in C} icm_{c,c'} \cdot QM_{c'} + \sum_{c' \in C} ice_{c,c'} \cdot QE_{c'}$	$c \in CT$	Trans-actions demand

Domestic and foreign trade

Table A.7 shows the equations for the domestic and foreign trade block. Equations TRD1-TRD3 are related to prices. In TRD1, the export price received by producers, PE_c, is defined as the world export price, transformed into domestic currency via the exchange rate and adjusted for export taxes and the transactions (trade and transport) cost per unit of exports; the unit transactions cost is defined as the product of an input coefficient ($ice_{c',c}$) and the input price, summed over all inputs. In analogous fashion, equation TRD2 defines the domestic currency import price for demanders, PM_c, on the basis of the world import price, the exchange rate, and import tariffs, in this case with the unit transactions cost added to the price; note that this price does not include the sales tax (cf. equation TRD7). In both equations, it is assumed that the modeled economy is small; thus, world prices for exports and imports (pwe_c and pwm_c) are exogenous. Equation TRD3 links the demander and supplier prices for domestic output sold domestically, PDD_c and PDS_c: the demander price is defined as the supplier price plus the transactions cost per unit of domestically sold output; as will be discussed below, either of these prices can be seen as the market-clearing price for this category of outputs (cf. equation SYS2).

Equations TRD4-TRD7 explain the allocation of domestic demands between imports and domestic purchases and define the related demander price of composite commodities, made up of imported and/or domestically produced commodities. Following the Armington assumption, we assume that products are differentiated on the basis of their origin (domestic or foreign). Consequently, it is possible to account for two-way trade (i.e., the same product may be both exported and imported; this is in distinction to merely accounting for net trade). The composite commodities, which are demanded domestically, are a CES aggregation of domestic and imported products (equation TRD4). For commodities that lack either imports or domestic production, TRD4 is replaced by TRD5: their composite quantities are simply identical

to the quantity of imports or the quantity of domestic sales of domestic output.[6] For commodities with both imports and domestic purchases of domestic output, equation TRD6 is the tangency condition that determines the domestic/imported mix; it is part of the first-order conditions for demander cost minimization. Equation TRD7 implicitly defines the price of the composite product, PQ_c, given the composite quantity, QQ_c and the sum of the values of imports and domestic purchases, adjusted upward for the sales tax; notice that the tax base excludes the tax.

Equations TRD8-TRD11 turn to the supply side, addressing the allocation of domestic output between two destinations: exports and domestic markets: output is viewed as imperfectly transformable between these two destinations. A CET (Constant Elasticity of Transformation) function defines the allocation of output, QX_c, between exports and domestic sales (QE_c and QD_c; equation TRD8).[7] For products that lack exports or domestic sales, equation TRD8 is replaced by equation TRD9: output quantity equals the export quantity or the domestic sales quantity. Equation TRD10 defines the ratio between exports and domestic sales for commodities that have both; it is part of the first-order conditions for producer profit maximization. Equation TRD11 implicitly defines the producer price for commodity *c*, PX_c, given the output quantity and, on the right-hand side, the sum of the values of domestic sales and exports at producer prices.

The last equation in this block, TRD12, defines the total transactions demand, i.e., the demand for commodities (in practice trade and transportation services) that are used in the distribution of commodities between (1) producers and the border (for exports); (2) producers and domestic demanders (for domestic sales); and (3) demanders and the border (for imports). For each category, the transactions demand is defined as the quantity that is distributed (exported, imported, or sold domestically) multiplied by exogenous unit input coefficients.

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[6] The elasticity of substitution between domestic purchases and imports is $\sigma_c^q = 1/(1+\rho_c^q)$.

[7] The elasticity of transformation between domestic sales and exports is $\sigma_c^x = 1/(\rho_c^x - 1)$.

Table A.8. Equations for domestic institutions

INS1	$YIF_{i,f} = shif_{i,f} \cdot YF_f(1 - tf_f)$	$i \in INS$ $f \in F$	Institutional factor income
INS2	$YI_i = \sum_f YIF_{i,f} + trnfr_{i,gov} \cdot \overline{CPI} + trnsfr_{i,row} \cdot EXR + \sum_{i' \in INSDNG} TRII_{i,i'}$	$i \in INSDNG$	Non-gov't institution income
INS3	$MPS_i = mpsb_i \cdot MPSSCAL$	$i \in INSDNG$	Marginal propensity to save
INS4	$SAVINS_i = MPS_i(1 - TY_i)YI_i$	$i \in INSDNG$	Non-gov't institution savings
INS5	$TRII_{i,i'} = shii_{i,i'}(1 - MPS_{i'})(1 - TY_{i'})YI_{i'}$	$i \in INS$ $i' \in INSDNG$	Institutional transfers
INS6	$EH_h = \left(1 - \sum_{i \in INS} shii_{i,h}\right)(1 - MPS_h)(1 - TY_h)YI_h$	$h \in H$	Household consumption expenditure
INS7	$QH_{c,h} = \gamma_{c,h} + \frac{\beta_{c,h}}{PQ_c}\left(EH_h - \sum_{c'} PQ_{c'} \cdot \gamma_{c',h}\right)$	$c \in C$ $h \in H$	Household consumption demand
INS8	$YG = \sum_{i \in INSDNG} TY_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{c \in C} tq_c\,(PM_c \cdot QM_c + PDD_c \cdot QD_c) + \sum_{c \in C} tm_c \cdot EXR \cdot pwm_c \cdot QM_c + \sum_{c \in C} te_c \cdot EXR \cdot pwe_c \cdot QE_c + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + EXR \cdot trnsfr_{gov,row} + \sum_{i \in INSDNG} TRII_{gov,i} + \sum_{f \in F} YIF_{gov,f}$		Government income
INS9	$TY_i = tyb_i \cdot TYSCAL$	$i \in INSDNG$	Direct tax rate
INS10	$QG_c = qgb_c \cdot \overline{QGSCAL}$	$c \in C$	Government consumption demand
INS11	$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{i,gov} \cdot \overline{CPI} + trnsfr_{row,gov} \cdot EXR$		Government expenditure
INS12	$SAVG = YG - EG$		Government savings

INS13	$\overline{RSAVG} = \frac{SAVG}{\overline{CPI}}$		Real government savings

Domestic institutions

The equations presented in Table A.8 define current incomes and their uses (consumption, savings, and transfers) for domestic institutions (government and non-government). The first equation, INS1, computes the factor income received by each institution *i* as the total income of the factor (YF_f) net of direct taxes on factor income multiplied the share of the institution in the total endowment of the factor ($shif_{i,f}$).

Equations INS2-INS7 apply to domestic non-government institutions (in the set *INSDNG*), invariably one or more RHs but, depending on the database, potentially also including enterprises and other institutions like NGOs. This set of equations defines their incomes, savings, and spending. As shown by equation INS2, the income of institution *i* is the sum of four elements: (1) factor income; (2) transfers from the government, indexed to the consumer price index (CPI); (3) transfers from rest of the world (i.e., remittances), exogenous in foreign currency and transformed into domestic currency; and (4) transfers from other domestic non-government institutions. Equation INS3 computes the marginal propensity to save as the exogenous base rate, $mpsb_i$, multiplied by a scaling factor, $MPSSCAL$, which here is exogenous; for the base solution, the scaling factor is equal to one but otherwise it may take on other values and, under alternative treatments of the savings-investment balance, it may be endogenous. Equation INS4 computes the value of savings for each institution as its total income net of direct taxes multiplied by its savings rate. Equation INS5 defines transfers from domestic non-government institutions to any other institution (government, non-government, or the rest of the world). These transfer flows are endogenous, defined as exogenous shares of the incomes of domestic non-government institutions net of direct taxes and savings. Equation INS6 computes the consumption spending by domestic non-government institutions as their

income net of transfers to other institutions, savings, and direct taxes.[8] Equation INS7 defines consumption expenditure for the subset of consuming units in *INSDNG*, labeled as households – the set *INSDNG* may also include institutions that do not consume, like enterprises. Household consumption expenditure is distributed across commodities on the basis of a Stone-Geary utility function from which linear expenditure system (LES) demand functions are derived.

A parallel set of equations, INS8-INS13, defines incomes, savings, and current spending for the government. Equation INS8 computes government current income as the sum of (1) revenues from direct taxes (on institutions and factors); (2) revenues from indirect taxes (taxes on sales, imports, exports, imports, and producer output value); (3) transfers from the rest of the world and domestic non-government institutions; and (4) factor income. Note that transfers from the rest of the world to the government, like private transfers, are exogenous in foreign currency and transformed into domestic currency. As indicated by Equation INS9, initial direct tax rates are scaled by the variable *TYSCAL*, the variable that clears the current government budget. Equation INS10 computes government consumption of *c* as the product of (a) qgb_c, which is exogenous and initially holds the base-year quantities; and (b) a scaling factor, *QGSCAL*, which here also is exogenous; for the base solution, this scaling factor is equal to one but otherwise it may take on other values and, under alternative treatments of the government budgetary balance, it may be endogenous. Government current spending (equation INS11) is defined as the sum of government consumption and government transfers (domestic and/or to the rest of the world). Nominal government savings is the difference between current income and current spending (equation INS12). *Real* government savings, the ratio between nominal government savings and the CPI (the numéraire), is fixed (equation INS13). [9]

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[8] Among the potential domestic non-government institutions, enterprises do not consume by definition whereas households consume. If the database includes NPISHs (non-profit institutions in the service of households, including NGOs), then these may also consume.

[9] The model would not be homogeneous (of degree zero) if GSAV were fixed; hence the need to introduce real government savings. Note that, for domestic *non*-government institutions, savings is expressed as a share of income net of direct taxes. By construction, their income is equal to the sum of their spending (consumption,

Table A.9. Equations for system constraint and price indices

SYS1	$\sum_{c\in C} pwe_c \cdot QE_c + \sum_{i\in INSD} trnsfr_{i,row} + \sum_{f\in F} trnsfr_{f,row} + \overline{SAVF} = \sum_{c\in C} pwm_c \cdot QM_c + trnsfr_{row,gov} + \sum_{i\in INSDNG} \frac{TRII_{row,i}}{EXR} + \sum_{f\in F} \frac{YIF_{row,f}}{EXR}$		Current account of the balance of payments
SYS2	$\sum_h QH_{c,h} + \sum_a QINT_{c,a} + QINV_c + QG_c + qdstk_c + QT_c = QQ_c$	$c \in C$	Composite commodity demand-supply balance
SYS3	$QINV_c = qinvb_c \cdot \overline{QINVSCAL}$	$c \in C$	Investment demand
SYS4	$\sum_{i\in INSDNG} SAVINS_i + SAVG + EXR \cdot \overline{SAVF} = \sum_{c\in C} PQ_c(QINV_c + qdstk_c) + WALRAS$		Savings-investment balance
SYS5	$\sum_{c\in C} PQ_c \cdot cwts_c = \overline{CPI}$		Consumer price index
SYS6	$\sum_{c\in C} PDS_c \cdot dwts_c = DPI$		Producer price index

System constraints and price indices

The last equation block, presented in Table A.9, defines system constraints (the current account of the balance of payments, commodity markets, and the savings-investment balance) and two aggregate price indices (the CPI and the producer price index for domestic sales), either of which may be the numéraire of the model. The constraints for factor markets were defined in the first equation block.

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savings, direct tax payments and transfers to other institutions). For the government, income and current spending are specified independently with savings as the difference. Other government closures are possible; for example, fixing *TYSCAL* and flexing *RGSAV* would turn government savings into the clearing variable for the government.

The rest of the world is represented by the current account of the balance of payments, expressed in foreign currency (equation SYS1). Except for the exchange rate, *EXR*, which is used to convert terms in domestic currency into foreign currency, the different terms in this balance were covered in the preceding blocks. The left-hand side shows the inflows of foreign exchange: it is the sum of exports, current transfers, factor payments, and foreign savings; the latter is an item that corresponds to the current-account deficit and has as its counterpart a surplus in the capital and financial account. On the right-hand side, foreign exchange outflows out due to imports, current transfers from the government and non-government institutions, and factor payments. As written, it is assumed that foreign savings are exogenous and that external balance is achieved via adjustments in the value of *EXR*. To illustrate, elimination of a balance of payments deficit would be achieved via depreciation (a higher value for *EXR*), which adjusts the real exchange rate, raising the *PE/PDS* ratio for suppliers and reducing the *PDD/PM* ratio for demanders, in this way bringing about higher exports and lower imports (cf. equations TRD1, TRD2, TRD6 and TRD10); through the same mechanisms, appreciation would eliminate a surplus. Alternatively, the exchange rate could be fixed (implicitly indexed to the model numéraire; see discussion below); if so, adjustment in foreign savings would clear the external balance.

Equation SYS2 is the condition for supply-demand equilibrium in commodity markets. The composite supply, made up of domestic and imported varieties, is used for household consumption, intermediate use, investment, government consumption, and changes in inventories. For the domestic component of the commodity market (where quantities demanded for and supplied of domestic output meet – the variable QD_c in equations TRD6 and TRD10, respectively), demanders respond to changes in PDD_c and suppliers to changes in PDS_c, two variables that are linked (equation TRD3). To illustrate, if this market has excess demand for a commodity *c*, an increase in PDS_c would increase quantities supplied (by generating incentives for higher production of *c* and allocation of a larger share of this output to domestic sales as opposed to exports) while the simultaneous increase in PDD_c, would reduce quantities demanded. As stated above, the small-country assumption applies to the

imported part of the commodity market: the quantities supplied are infinitely elastic at exogenous international prices and could be viewed as clearing this commodity market constraint.

The only term in equation SYS2 that had not yet been covered, investment demand (by origin), $QINV_c$, is defined in equation SYS3, which computes investment demand for *c* as the product of (a) $qinvb_c$, which is exogenous and initially holds the base-year quantities; and (b) a scaling factor, *QINVSCAL*. Like *QGSCAL*, *QINVSCAL* is exogenous and equal to one for the base solution. In other simulations it may take on other values and, depending on the treatment of the savings-investment balance, the next equation, it may be endogenous.

In the savings-investment balance, SYS4, the left-hand side defines available savings as the sum of savings from domestic non-government institutions, the government, and the rest of the world; the right-hand side defines the total investment value.[10] The variable *WALRAS* must be zero in equilibrium (if not, savings and investment are not equal and the model did not solve correctly). In our application, this balance is cleared by adjustments in the variable $MPSSCAL$, which is endogenous (see equation INS4). In order to switch from savings-driven investment to investment-driven savings, the analyst may exogenize $MPSSCAL$ and endogenize $INVSCAL$ (in equation SYS3). Finally, equation SYS5 defines the *CPI*, which is exogenous and the model numéraire, as a weighted average of composite commodity prices (PQ_c); the weights are the shares of each commodity in private (i.e., household) consumption.[11]

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[10] Given that the model is static, there is no need to disaggregate investment into different types (for example government and non-government) or to address how different types of investment are financed.

[11] Under an alternative closure for the balance of payments, the analyst may instead wish to fix the real exchange rate, which typically is defined as the ratio between the nominal exchange rate and an index of domestic producer prices; if so, it would be necessary to augment the model with this variable and an equation in which it is defined.